FORMULAS FOR REFERENCE

SPHERE	Surface area	$= 4\pi r^2$
	Volume	$= \frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	$= 2\pi rh$
	Volume	$= \pi r^2 h$
CONE	Area of curved surface	$= \pi rl$
	Volume	$= \frac{1}{3}\pi r^2 h$
PRISM	Volume	= base area × height
PYRAMID	Volume	$= \frac{1}{3}$ × base area × height

SECTION A (51 marks)
Answer ALL questions in this section.
There is no need to start each question on a fresh page.

1. Factorize

   (a) $x^2 - 9$,

   (b) $ac + bc - ad - bd$.

   (4 marks)

2. Simplify

   (a) $\sqrt{27} - \sqrt{12}$,

   (b) $\dfrac{1}{2\sqrt{3} + \sqrt{2}}$.

   (5 marks)

3. (a) Simplify $\dfrac{x^3 y^2}{x^{-3} y}$ and express your answer with positive indices.

   (b) Simplify $\dfrac{\log 8 + \log 4}{\log 16}$.

   (5 marks)

4. Solve (i) $2x - 17 > 0$,

   (ii) $x^2 - 16x + 63 > 0$.

   Hence write down the range of values of x which satisfy both the inequalities in (i) and (ii).

   (5 marks)

5. In Figure 1, ABC is a right-angled triangle. $AB = 3$, $BC = 4$, $CD = 6$, $\angle ABC = 90°$ and $\angle ACD = 60°$. Find

(a) AC,

(b) AD,

(c) the area of ΔACD.

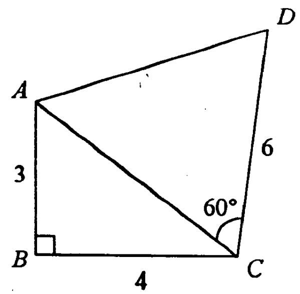

Figure 1

(5 marks)

6. In Figure 2, the bearings of two ships A and B from a lighthouse L are 020° and 110° respectively. B is 20 km and at a bearing of 140° from A. Find

(a) the distance of L from B,

(b) the bearing of L from B.

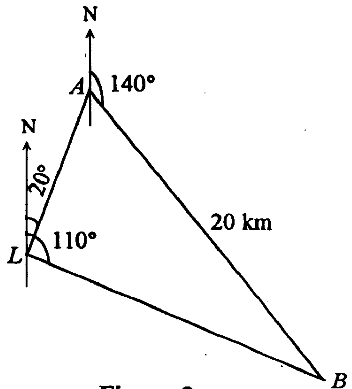

Figure 2

(5 marks)

7. The ratio of the volumes of two similar solid circular cones is 8 : 27.

(a) Find the ratio of the height of the smaller cone to the height of the larger cone.

(b) If the cost of painting a cone varies as its total surface area and the cost of painting the smaller cone is $ 32, find the cost of painting the larger cone.

(4 marks)

8. The roots of the equation $2x^2 - 7x + 4 = 0$ are α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$.

(b) Find the quadratic equation whose roots are $\alpha + 2$ and $\beta + 2$.

(6 marks)

9. In Figure 3, AC is a diameter of the circle. $AC = 4$ cm and $\angle BAC = 30°$. Find

(a) $\angle BDC$ and $\angle ADB$,

(b) $\overset{\frown}{AB} : \overset{\frown}{BC}$,

(c) $AB : BC$.

(You are not required to give reasons.)

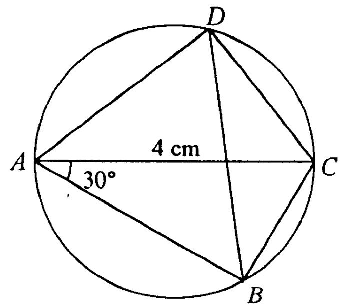

Figure 3

(6 marks)

10. Suppose the population of a town grows by 2% each year and its population at the end of 1996 was 300 000.

(a) Find the population at the end of 1998.

(b) At the end of which year will the population just exceed 330 000?

(6 marks)

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SECTION B (48 marks)
Answer any FOUR questions in this section.
Each question carries 12 marks.

11. The following are the marks scored by a class of 35 students in a Mathematics test:

0	0	5	8	11	12	41	42	45	48
50	62	70	73	73	73	77	78	80	80
82	82	82	83	83	85	85	87	90	90
95	95	95	95	98					

(a) Find the mean, mode, median and standard deviation of the above marks. (Working need not be shown.)

(4 marks)

(b) Explain briefly why the mean may not be a suitable measure of central tendency of the distribution of marks in the Mathematics test.

(1 mark)

(c) The mean and standard deviation of the marks scored by the same class of students in an English test are 63 and 15 respectively.

(i) The standard score of a student in the English test was 0.4. Find the mark the student scored in this test.

(ii) Assume that the marks in the English test are normally distributed and the marks scored by Lai Wah in both the Mathematics and English tests are 78.

(I) What percentage of her classmates scored fewer marks than Lai Wah in the Mathematics test?

(II) Relative to her classmates, did Lai Wah perform better in the English test than in the Mathematics test?

(iii) The English teacher later found that a student was given 10 marks fewer in the English test. Find the mean of the marks in the English test after the wrong mark has been corrected.

(7 marks)

12. Figure 4.1 shows a greenhouse $VABCD$ in the shape of a right pyramid with a square base of side 6 m. M is the mid-point of BC and VN is the height of the pyramid. Each of the triangular faces makes an angle θ with the square base.

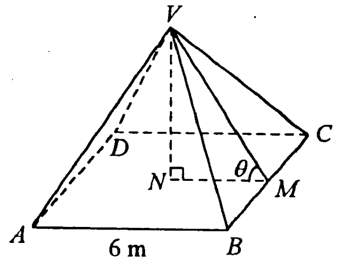

Figure 4.1

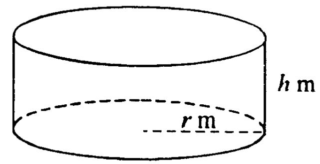

Figure 4.2

(a) (i) Express VN and VM in terms of θ.

(ii) Find the capacity and total surface area of the greenhouse (excluding the base) in terms of θ.

(5 marks)

(b) Figure 4.2 shows another greenhouse in the shape of a right cylinder with base radius r m and height h m. It is known that both the base areas and the capacities of the two greenhouses are equal.

(i) Express r in terms of π.

(ii) Express h in terms θ.

(iii) If the total surface areas of the two greenhouses (excluding the bases) are equal, show that

$$3+\sqrt{\pi}\tan\theta=\frac{3}{\cos\theta} \qquad \text{...............(*)}.$$

(iv) Show that equation (*) has a root between 61° and 62°.

(7 marks)

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13. Miss Lee makes and sells handmade leather belts and handbags. She finds that if a batch of x belts is made, where $1 \le x \le 11$, the cost per belt $\$B$ is given by $B = x^2 - 20x + 120$. Page 8 shows the graph of the function $y = x^2 - 20x + 120$.

(a) Use the given graph to write down the number(s) of belts in a batch that will make the cost per belt

(i) a minimum,

(ii) less than $90.

(3 marks)

(b) Miss Lee also finds that if a batch of x handbags is made, where $1 \le x \le 8$, the cost per handbag $\$H$ is given by $H = x^2 - 17x + c$ (c is a constant). When a batch of 3 handbags is made, the cost per handbag is $144.

(i) Find c.

(ii) By adding a suitable straight line on the given graph, find the number of handbags in a batch that will make the cost per handbag $120.

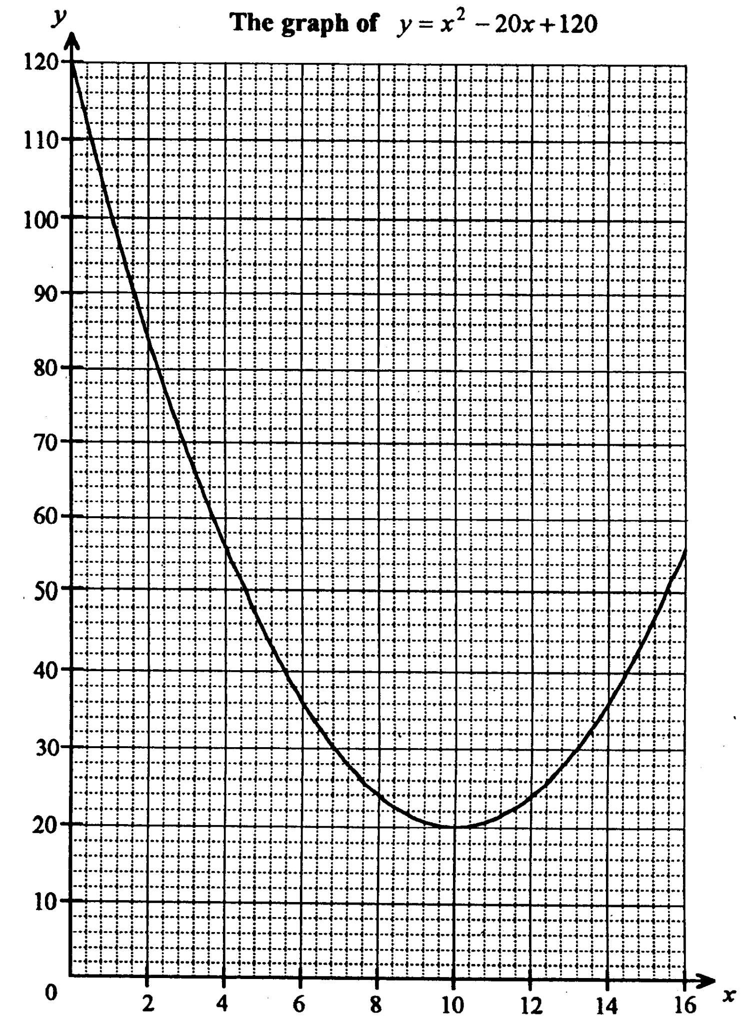
(iii) Miss Lee made a batch of 10 belts and a batch of 6 handbags. She managed to sell 6 belts at $100 each and 4 handbags at $300 each while the remaining belts and handbags sold at half of their respective cost. Find her gain or loss.

(9 marks)

Candidate Number	Centre Number	Seat Number	Page Total

13.(Cont'd) **If you attempt Question 13, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**

The graph of $y = x^2 - 20x + 120$



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14. In a small pond, there were exactly 40 *small* fish and 10 *large* fish. The ranges of their weights W g are shown in the following table:

	Weight (W g)
Small fish	$0 < W \leq 100$
Large fish	$500 \leq W \leq 600$

In the morning on a certain day, a man went fishing in the pond. He caught two fish and their total weight was T g. Suppose each fish was equally likely to be caught.

(a) Find the probability that

(i) $0 < T \leq 200$,

(ii) $500 \leq T \leq 700$,

(iii) $1000 \leq T \leq 1200$,

(iv) $T > 1200$.

(8 marks)

(b) Suppose the two fish he caught in the morning were returned alive to the pond. He went fishing again in the pond in the afternoon and also caught two fish.

(i) If the total weight of the fish caught in the morning was 650 g , find the probability that the difference between the total weights of the fish caught in the morning and in the afternoon is more than 200 g .

(ii) Find the probability that the difference between the total weights of the fish caught in the morning and in the afternoon is more than 200 g .

(4 marks)

Go on to the next page

15. On Page 12, figure A_1 is a square of side ℓ. To the middle of each of three sides of figure A_1, a square of side $\frac{\ell}{3}$ is added to give figure A_2.

Following the same pattern, squares of side $\frac{\ell}{9}$ are added to figure A_2 to give figure A_3. The process is repeated indefinitely to give figures A_4, A_5, $\ldots$, A_n, $\ldots$.

(a) (i) Table 1 on Page 12 shows the numbers and the lengths of sides of the squares added when producing A_2 from A_1, A_3 from A_2 and A_4 from A_3. Complete Table 1.

(ii) Find the total area of all the squares in A_4.

(iii) As n increases indefinitely, the total area of all the squares in A_n tends to a constant k. Express k in terms of ℓ.

(7 marks)

(b) The overlapping line segments in figures A_1, A_2, A_3, $\ldots$, A_n, $\ldots$ are removed to form figures B_1, B_2, B_3, $\ldots$, B_n, $\ldots$ as shown on Page 12.

(i) Complete Table 2 on Page 12.

(ii) Write down the perimeter of B_n.

What would the perimeter of B_n become if n increases indefinitely?

(5 marks)

Candidate Number	Centre Number	Seat Number	Page Total

15.(Cont'd) **If you attempt Question 15, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**

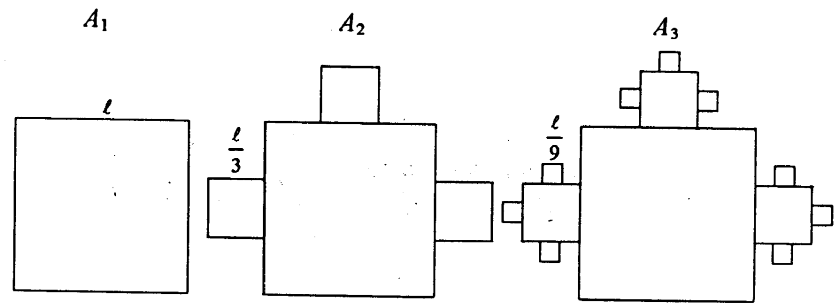

Table 1

	$A_1 \to A_2$	$A_2 \to A_3$	$A_3 \to A_4$
Number of squares added	3	9	
Length of sides of the squares added	$\frac{\ell}{3}$	$\frac{\ell}{9}$	

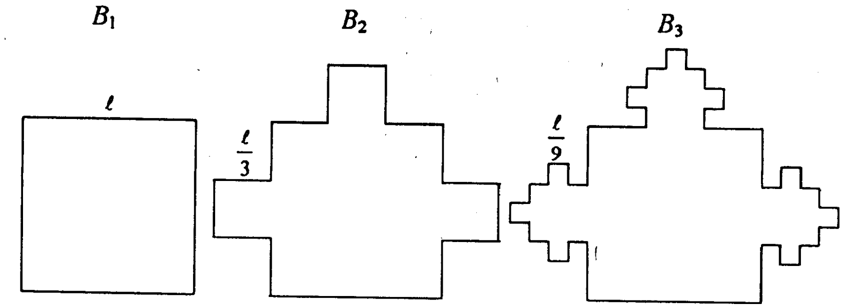

Table 2

	B_1	B_2	B_3	B_4
Perimeter	4ℓ			

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16.

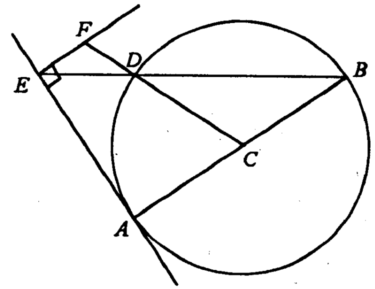

Figure 5.1

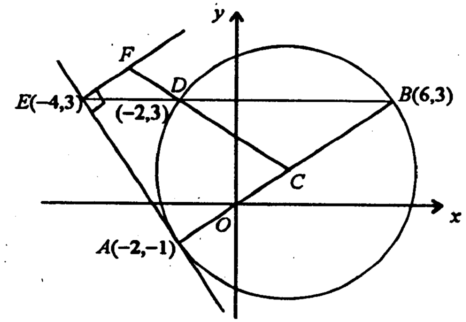

Figure 5.2

(a) In Figure 5.1, D is a point on the circle with AB as diameter and C as the centre. The tangent to the circle at A meets BD produced at E. The perpendicular to this tangent through E meets CD produced at F.

(i) Prove that $AB // EF$.

(ii) Prove that $FD = FE$.

(iii) Explain why F is the centre of the circle passing through D and touching AE at E.

(8 marks)

(b) A rectangular coordinate system is introduced in Figure 5.1 so that the coordinates of A and B are $(-2, -1)$ and $(6, 3)$ respectively. It is found that the coordinates of D and E are $(-2, 3)$ and $(-4, 3)$ respectively as shown in Figure 5.2. Find the coordinates of F.

(4 marks)

END OF PAPER