2008 Mathematics

評卷參考 *

Marking Scheme

* 此部分只設英文版本

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

6. Marks may be deducted for wrong units (u) or poor presentation (pp).

   a. The symbol (u-*1*) should be used to denote 1 mark deducted for u. At most deduct ***1 mark*** for u in Section A. Do not deduct any marks for u in Section B.

   b. The symbol (pp-*1*) should be used to denote 1 mark deducted for pp. At most deduct ***1 mark*** for pp in each of Section A and Section B. For similar pp, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in the paper for the same pp.

   c. At most deduct 1 mark in each question. Deduct the mark for u first if both marks for u and pp may be deducted in the same question.

   d. In any case, do not deduct any marks for pp or u in those steps where candidates could not score any marks.

7. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

* 此部分只設英文版本

Paper 1

Solution	Marks	Remarks
1. $\dfrac{(ab)^3}{a^2}$		
$=\dfrac{a^3b^3}{a^2}$	1M	for $(xy)^m = x^m y^m$
$=a^{3-2}b^3$	1M	for $\dfrac{x^m}{x^n} = x^{m-n}$
$=ab^3$	1A	
	----------(3)	
2. (a) $\dfrac{14x}{5} \geq 2x+7$		
$14x \geq 5(2x+7)$		
$14x \geq 10x+35$		
$14x-10x \geq 35$	1M	for putting x on one side
$4x \geq 35$		
$x \geq \dfrac{35}{4}$	1A	$x \geq 8.75$
$\dfrac{14x}{5} \geq 2x+7$		
$\dfrac{14x}{5} - 2x \geq 7$	1M	for putting x on one side
$\dfrac{4x}{5} \geq 7$		
$x \geq \dfrac{35}{4}$	1A	$x \geq 8.75$
(b) The required least integer is 9 .	1A	
	----------(3)	
3. (a) The values of m are 1 and 3 .	1A + 1A	1A for either + 1A for all
(b) The values of k are 2 and 3 .	1A	for all correct
	----------(3)	
4. $\sin\angle PQR = \dfrac{PR}{PQ}$	1M	pp−1 for any undefined symbol
$\sin\angle PQR = \dfrac{9}{14}$		
$\angle PQR \approx 40.00520088°$		
$\angle PQR \approx 40.0°$	1A	r.t. 40°
Thus, the bearing of Q from P is S40.0°W .	1M	accept S40°W , 220 or 220°
$\cos\angle QPR = \dfrac{PR}{PQ}$	1M	pp−1 for any undefined symbol
$\cos\angle QPR = \dfrac{9}{14}$		
$\angle QPR \approx 49.99479912°$		
$\angle QPR \approx 50.0°$	1A	r.t. 50°
Thus, the bearing of Q from P is S40.0°W .	1M	accept S40°W , 220 or 220°
	----------(3)	

Solution	Marks	Remarks
5. The required probability		
$=\frac{2}{6}$	1A + 1A	1A for numerator + 1A for denominator
$=\frac{1}{3}$	1A	r.t. 0.333
	----------(3)	
6. (a) $\frac{2s+t}{s+2t}=\frac{3}{4}$		
$4(2s+t)=3(s+2t)$	1M	
$8s+4t=3s+6t$		
$5s=2t$		
$t=\frac{5s}{2}$	1A	
Let $r=\frac{t}{s}$.		
$\frac{2s+t}{s+2t}=\frac{3}{4}$		
$\frac{2+r}{1+2r}=\frac{3}{4}$		
$4(2+r)=3(1+2r)$	1M	
$8+4r=3+6r$		
$2r=5$		
$r=\frac{5}{2}$		
$t=\frac{5s}{2}$	1A	
(b) $s+t=959$		
$s+\frac{5s}{2}=959$	1M	
$\frac{7s}{2}=959$		
Thus, we have $s=274$ and $t=685$.	1A	for both correct
By (a), let $s=2k$ and $t=5k$, where k is a non-zero constant.		
$s+t=959$		
$2k+5k=959$	1M	
$7k=959$		
$k=137$		
Thus, we have $s=274$ and $t=685$.	1A	for both correct
	----------(4)	

Solution	Marks	Remarks
7. (a) The estimated total amount		
$=(9)(4)+(17)(3)+(5)(2)$	1M + 1A	1M for either correct + 1A for all
$=36+51+10$		
$=\$97$	1A	u–1 for missing unit
(b) Note that the actual total amount is less than $97 .		
Thus, John has enough money to buy all the items needed.	1A	f.t.
	----------(4)	
8. (a) The number of girls in the school		
$=625(1-28\%)$	1M	
$=450$	1A	
(b) (i) The required percentage		
$=\left(\frac{860}{450+625}\right)(100\%)$	1M	accept without 100 %
$=\left(\frac{860}{1075}\right)(100\%)$		
$=80\%$	1A	
(ii) By (b)(i), we have $x=80$.	1A	
$625(80\%)+450(x\%)=860$		
$500+\frac{9x}{2}=860$		
$\frac{9x}{2}=360$		
$x=80$	1A	
	----------(5)	
9. Note that $x=\angle DAB$.		
Thus, we have $x=33^\circ$.	1A	u–1 for missing unit
Also note that $y=x+43^\circ$.	1M	can be absorbed
Thus, we have $y=76^\circ$.	1A	u–1 for missing unit
Since $\angle ACE=y$,	1M	can be absorbed
we have $\angle ACE=76^\circ$.		
Further note that $\angle ACE+y+z=180^\circ$.		
So, we have $76^\circ+76^\circ+z=180^\circ$.		
Thus, we have $z=28^\circ$.	1A	u–1 for missing unit
	----------(5)	

Solution	Marks	Remarks
10. (a) a		
$= 4$	1A	
b		
$= 37 - 12 - 4$	1M	can be absorbed (either one)
$= 21$	1A	
c		
$= 50 - 4 - 12 - 21 - 10$		
$= 3$	1A	
	----------(4)	
(b) The mean		
$= \frac{(2.7)(4)+(3.0)(12)+(3.3)(21)+(3.6)(10)+(3.9)(3)}{50}$	1M	can be absorbed
$= 3.276$ kg	1A	r.t. 3.28 kg u−1 for missing unit
The standard deviation		
≈ 0.299038459		
≈ 0.299 kg	1A	r.t. 0.299 kg u−1 for missing unit
	----------(3)	

Solution	Marks	Remarks
11. (a) $f(4)=9$	1M	
$4^2+4b-15=9$		
$4b=8$		
$b=2$	1A	
$f(x)=0$		
$x^2+2x-15=0$		
$(x-3)(x+5)=0$		
$x=3$ or $x=-5$	1A	for both correct
Thus, the two x-intercepts are 3 and -5 .		
	----------(3)	
(b) $f(x)=k$		
$x^2+2x-15=k$		
$x^2+2x-(15+k)=0$		
$\Delta=2^2-4(1)(-15-k)$	1M + 1A	
$=64+4k$		
Since $f(x)=k$ has two distinct real roots,		
$64+4k>0$	1M	for $\Delta>0$
$k>-16$	1A	
By (a), the x-coordinate of the vertex of the graph of $y=f(x)$ is -1 .	1M	
$f(-1)$		
$=(-1)^2+2(-1)-15$		
$=-16$	1A	
Since the coefficient of the x^2 term in $f(x)$ is positive, the graph of $y=f(x)$ opens upwards.		
Therefore, the least value of $y=f(x)$ is -16 .	1M	
Thus, we have $k>-16$.	1A	
	----------(4)	
(c) The equation of a required straight line is $y=-16$.	1A	accept $x=c$
	----------(1)	

Solution	Marks	Remarks
12. (a) B $=(-3,4)$	1A	pp−1 for missing '(' or ')'
C $=(4,-3)$	1A	pp−1 for missing '(' or ')'
	----------(2)	
(b) The slope of OB $=\frac{4-0}{-3-0}=\frac{-4}{3}$	1M	for using slope formula -----
The slope of OC $=\frac{-3-0}{4-0}=\frac{-3}{4}$		------------------------------either one
The slope of BC $=\frac{4-(-3)}{-3-4}=-1$		
Therefore, the slope of OB is not equal to the slope of OC.	1M	for comparing slopes
Thus, O, B and C are not collinear.	1A	f.t.
$OB=\sqrt{(-3-0)^2+(4-0)^2}=5$	1M	for using distance formula ---
$OC=\sqrt{(4-0)^2+(-3-0)^2}=5$		------------------------------either one
$BC=\sqrt{(-3-4)^2+(4-(-3))^2}=\sqrt{98}$		
Therefore, $OB+OC=10\neq\sqrt{98}=BC$.	1M	for comparing $OB+OC$ with BC
Thus, O, B and C are not collinear.	1A	f.t.
	----------(3)	
(c) The slope of BC $=\frac{4-(-3)}{-3-4}=-1$		
The slope of CD $=\frac{-1}{-1}=1$		
The equation of CD is		
$y-(-3)=1(x-4)$	1M	for point-slope form
$x-y-7=0$	1A	or equivalent
Since A is translated horizontally to D, the y-coordinate of D is 3.	1M	
Putting $y=3$ in $x-y-7=0$, we have $x=10$.		
Thus, the coordinates of D are $(10,3)$.	1A	pp−1 for missing '(' or ')'
The slope of BC $=\frac{4-(-3)}{-3-4}=-1$		
The slope of CD $=\frac{-1}{-1}=1$		
The equation of CD is		
$y-(-3)=1(x-4)$	1M	for point-slope form
$x-y-7=0$	1A	or equivalent
Since A is translated horizontally to D, the y-coordinate of D is 3.	1M	
Let the coordinates of D be $(a,3)$.		
Note that the product of the slope of BC and the slope of CD is -1.		
Therefore, we have $(-1)\left(\frac{3-(-3)}{a-4}\right)=-1$.		
So, we have $a=10$.	1A	
Thus, the coordinates of D are $(10,3)$.		pp−1 for missing '(' or ')'
	----------(4)	

Solution	Marks	Remarks
13. (a) Let r cm and h cm be the base radius and the height of X respectively.		
$2\pi r = 2\pi(OA)\left(\frac{216^\circ}{360^\circ}\right)$	1M	
$2\pi r = 2\pi(20)\left(\frac{216^\circ}{360^\circ}\right)$		
$r = 12$	1A	
$h = \sqrt{OA^2 - r^2}$	1M	
$h = \sqrt{20^2 - 12^2}$		
$h = 16$	1A	
Thus, the base radius and the height are 12 cm and 16 cm respectively.		u–1 for missing unit
	----------(4)	
(b) The volume of X		
$= \frac{1}{3}\pi(12^2)(16)$	1M	
$= 768\pi \text{ cm}^3$	1A	u–1 for missing unit
	----------(2)	
(c) Let s cm be the base radius of Y.		
$2\pi s = 2\pi(10)\left(\frac{108^\circ}{360^\circ}\right)$		
$s = 3$		
$\frac{\text{The slant height of } Y}{\text{The slant height of } X} = \frac{10}{20} = \frac{1}{2}$	1M	for finding ratio — either one
$\frac{\text{The base radius of } Y}{\text{The base radius of } X} = \frac{3}{12} = \frac{1}{4}$		
$\frac{\text{The base radius of } Y}{\text{The base radius of } X} \neq \frac{\text{The slant height of } Y}{\text{The slant height of } X}$	1M	for comparing 2 ratios
Thus, X and Y are not similar.	1A	f.t.
Let s cm and k cm be the base radius and the height of Y respectively.		
$2\pi s = 2\pi(10)\left(\frac{108^\circ}{360^\circ}\right)$		
$s = 3$		
$k = \sqrt{10^2 - 3^2}$		
$k = \sqrt{91}$		
The volume of $Y = \frac{1}{3}\pi(3^2)(\sqrt{91}) = 3\sqrt{91}\pi$		
$\frac{\text{The slant height of } Y}{\text{The slant height of } X} = \frac{10}{20} = \frac{1}{2}$	1M	for finding ratio — either one
$\left(\frac{\text{The slant height of } Y}{\text{The slant height of } X}\right)^3 = \frac{1}{8}$		
$\frac{\text{The volume of } Y}{\text{The volume of } X} = \frac{3\sqrt{91}\pi}{768\pi} = \frac{\sqrt{91}}{256}$		
$\frac{\text{The volume of } Y}{\text{The volume of } X} \neq \left(\frac{\text{The slant height of } Y}{\text{The slant height of } X}\right)^3$	1M	for comparing
Thus, X and Y are not similar.	1A	f.t.

Solution	Marks	Remarks
The curved surface area of X $= \pi(20)^2\left(\frac{216^\circ}{360^\circ}\right)$ $= 240\pi$ The curved surface area of Y $= \pi(10)^2\left(\frac{108^\circ}{360^\circ}\right)$ $= 30\pi$		
$\frac{\text{The slant height of } Y}{\text{The slant height of } X} = \frac{10}{20} = \frac{1}{2}$	1M	for finding ratio (either one)
$\left(\frac{\text{The slant height of } Y}{\text{The slant height of } X}\right)^2 = \frac{1}{4}$		
$\frac{\text{The curved surface area of } Y}{\text{The curved surface area of } X} = \frac{30\pi}{240\pi} = \frac{1}{8}$		
$\frac{\text{The curved surface area of } Y}{\text{The curved surface area of } X} \neq \left(\frac{\text{The slant height of } Y}{\text{The slant height of } X}\right)^2$	1M	for comparing
Thus, X and Y are not similar.	1A	f.t.
Since $\angle DPF \neq 216^\circ$,	1M	accept $\angle DPF \neq \angle AOC$
sectors $OABC$ and $PDEF$ are not similar.	1M	either one
$\frac{\text{The area of sector } OABC}{\text{The area of sector } PDEF} \neq \left(\frac{\text{The radius of sector } OABC}{\text{The radius of sector } PDEF}\right)^2$		
$\frac{\text{The curved surface area of } Y}{\text{The curved surface area of } X} \neq \left(\frac{\text{The slant height of } Y}{\text{The slant height of } X}\right)^2$		
Thus, X and Y are not similar.	1A	f.t.
	---------(3)	

Solution	Marks	Remarks
14. (a) (i) The required probability		
$=\frac{9}{15}$	1M	for either numerator or denominator correct
$=\frac{3}{5}$	1A	0.6
The required probability		
$=\dfrac{\frac{9}{36}}{\frac{15}{36}}$	1M	for either numerator or denominator correct
$=\frac{3}{5}$	1A	0.6
(ii) (1) The required probability		
$=2\left(\frac{8}{36}\right)\left(\frac{15}{35}\right)$	1M	for $(\frac{s}{m})(\frac{t}{m-1})$, $s<m$ and $t<m-1$
$=\frac{4}{21}$	1A	r.t. 0.190
(2) The required probability		
$=\frac{4}{21}+2\left(\left(\frac{8}{36}\right)\left(\frac{13}{35}\right)+\left(\frac{13}{36}\right)\left(\frac{15}{35}\right)\right)$	1M	for (a)(ii)(1)$+2\left((\frac{p}{n})(\frac{r}{n-1})+(\frac{r}{n})(\frac{q}{n-1})\right)$, $p<n$, $q<n-1$ and $r<n-1$
$=\frac{419}{630}$	1A	r.t. 0.665
The required probability		
$=1-\left(\left(\frac{8}{36}\right)\left(\frac{7}{35}\right)+\left(\frac{15}{36}\right)\left(\frac{14}{35}\right)+\left(\frac{13}{36}\right)\left(\frac{12}{35}\right)\right)$	1M	for $1-\left(\left(\frac{p}{n}\right)\left(\frac{p-1}{n-1}\right)+\left(\frac{q}{n}\right)\left(\frac{q-1}{n-1}\right)+\left(\frac{r}{n}\right)\left(\frac{r-1}{n-1}\right)\right)$, $p<n$, $q<n$ and $r<n$
$=\frac{419}{630}$	1A	r.t. 0.665
	----------(6)	
(b) (i) The median		
$=\$5\,000$	1A	
The inter-quartile range		
$=6\,400-4\,300$	1M	
$=\$2\,100$	1A	
(ii) An extra $1 000 can be given to each of the 36 salesgirls.	1M + 1A	Accept other answers satisfying (i) increase in each suggested bonus (ii) 20% increase in median (iii) no change in inter-quartile range 1M for any two conditions satisfied + 1A for all the conditions satisfied
	----------(5)	

Solution	Marks	Remarks
15. (a) $\angle BHD = 50° - 35° = 15°$ and $\angle BDH = 180° - 50° = 130°$		
By sine formula, we have		
$\dfrac{BH}{\sin \angle BDH} = \dfrac{DB}{\sin \angle BHD}$	1M	accept using tangent ratio twice
$\dfrac{BH}{\sin 130°} = \dfrac{50}{\sin 15°}$		
$BH \approx 147.9884223$		
$BH \approx 148$ m	1A	r.t. 148 m
Thus, the distance between B and H is 148 m .		
	----------(2)	
(b) (i) By cosine formula, we have		
$\cos \angle CBH = \dfrac{BC^2 + BH^2 - CH^2}{2(BC)(BH)}$	1M	accept using Pythagoras' theorem twice
$\cos \angle CBH \approx \dfrac{210^2 + 147.9884223^2 - 130^2}{2(210)(147.9884223)}$		
$\angle CBH \approx 37.81747348°$		
$\angle CBH \approx 37.8°$	1A	r.t. 37.8°
Let $s = \dfrac{1}{2}(BC + CH + BH)$. Then, we have $s \approx 243.9942112$ m .		
$\dfrac{1}{2}(BC)(BH)\sin \angle CBH = \sqrt{s(s-BC)(s-CH)(s-BH)}$	1M	
$\dfrac{1}{2}(210)(147.9884223)\sin \angle CBH \approx \sqrt{s(s-210)(s-130)(s-147.9884223)}$		
$\angle CBH \approx 37.81747348°$		
$\angle CBH \approx 37.8°$	1A	r.t. 37.8°
(ii) Let E be the point on BC such that $HE \perp BC$ and $AE \perp BC$.		
The required angle is $\angle AEH$.	1A	for identifying the required angle
$EH = BH \sin \angle CBH$	1M	
$\approx 147.9884223 \sin 37.81747348°$		
≈ 90.73880495		
$AH = BH \sin \angle ABH$	1M	
$\approx 147.9884223 \sin 35°$		
≈ 84.88267191		
$\sin \angle AEH = \dfrac{AH}{EH}$	1M	for finding the required angle
$\sin \angle AEH = \dfrac{BH \sin \angle ABH}{BH \sin \angle CBH}$		
$\sin \angle AEH \approx \dfrac{\sin 35°}{\sin 37.81747348°}$		
$\angle AEH \approx 69.30285561°$		
$\angle AEH \approx 69.3°$	1A	r.t. 69.3°
Thus, the required angle is 69.3° .		
(iii) Since HE is the line of the greatest slope of the plane BCH , the greatest angle of elevation of H from a point on BC is $\angle AEH$.	1M	
Note that $\angle AEH < 75°$.		
Thus, it is impossible for Christine to find a point K on BC such that the angle of elevation of H from K is 75° .	1M	for drawing conclusion by using (b)(ii)
	----------(9)	

Solution	Marks	Remarks
16. (a) Let d be the common difference.		
Then, we have $a+d=10$ and $a+3d=24$.	1M	for either, accept $b=\frac{10+24}{2}$
Solving, we have $d=7$.		
a $=3$	1A	
b $=3+(2)(7)$ $=17$	1A	
	----------(3)	
(b) (i) The amount of salaries tax charged $=\$0.2P$	1A	
(ii) Let $\$P$ be the net total income of the citizen.		
If the citizen has to pay salaries tax at the standard rate, we have $(30\,000)(3\%)+(30\,000)(10\%)+(30\,000)(17\%)+(P-172\,000-90\,000)(24\%)\geq 0.2P$	1M + 1A	1M for the sum of a number of terms in the left hand side with any one term correct
$0.24P-53\,880\geq 0.2P$		
$P\geq 1\,347\,000$		
Thus, the least net total income of the citizen is $\$1\,347\,000$.	1A	
	----------(4)	
(c) By (b)(ii), Peter has to pay his salaries tax at the standard rate.		
The amount of salaries tax charged $=(0.2)(1\,400\,000)$ (by (b)(i))	1M	
$=\$\,280\,000$		
The amount $=(23\,000)\left(1+\frac{3}{12}\%\right)^{12}+(23\,000)\left(1+\frac{3}{12}\%\right)^{11}+\cdots+(23\,000)\left(1+\frac{3}{12}\%\right)$	1M	
$=\frac{(23\,000)(1+0.25\%)\left((1+0.25\%)^{12}-1\right)}{(1+0.25\%)-1}$	1M	for the sum of geometric sequence
$\approx 280\,526.3706$		
$>280\,000$		
Thus, Peter will have enough money to pay his salaries tax on the due day.	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
17. (a) $\angle BAP = \angle CAP$ (in-centre of Δ)		[Δ內心]
$BP = CP$ (equal $\angle$s , equal chords)		[等角對等弦]
$\angle BCP = \angle CAP$ (equal chords, equal $\angle$s)		[等弦對等角]
Join CI.		
$\angle CIP$		
$= \angle CAP + \angle ACI$ (ext. $\angle$ of Δ)		[Δ外角]
$= \angle BCP + \angle BCI$ (in-centre of Δ)		[Δ內心]
$= \angle ICP$		
So, we have $CP = IP$. (sides opp. equal $\angle$s)		[等角對邊相等] [底角相等] [等角對等邊] [等腰Δ底角(等)的逆理]
Thus, we have $BP = CP = IP$.		
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	3	
Case 2 Any correct proof without reasons.	2	
Case 3 Incomplete proof with any one correct step and one correct reason.	1	
	----------(3)	
(b) (i) By (a), the required circle passes through B , C and I .		
Let the equation of the required circle be $x^2 + y^2 + Dx + Ey + F = 0$.		
$\because$ the coordinates of B and C are $(-80, 0)$ and $(64, 0)$ respectively		
$\therefore \begin{cases} (-80)^2 + (0)^2 + D(-80) + E(0) + F = 0 \\ (64)^2 + (0)^2 + D(64) + E(0) + F = 0 \end{cases}$	1M	either one
So, we have $D = 16$ and $F = -5120$.	1A	for both correct
$\because$ the coordinates of I are $(0, 32)$.		
$\therefore$ $(0)^2 + (32)^2 + D(0) + E(32) + F = 0$		
So, we have $E = 128$.	1A	
Thus, the required equation is $x^2 + y^2 + 16x + 128y - 5120 = 0$.		$(x+8)^2 + (y+64)^2 = 9280$
Let the coordinates of P be (a, b) .		
Since $BP = CP = IP$, we have		
$\begin{cases} (-80-a)^2 + (0-b)^2 = (64-a)^2 + (0-b)^2 \\ (-80-a)^2 + (0-b)^2 = (0-a)^2 + (32-b)^2 \end{cases}$	1M	for either one
Solving, we have $a = -8$ and $b = -64$.	1A	for both correct
The equation of the required circle is		
$(x-(-8))^2 + ((y-(-64))^2 = (64-(-8))^2 + (0-(-64))^2$		
Thus, the required equation is $(x+8)^2 + (y+64)^2 = 9280$.	1A	$x^2 + y^2 + 16x + 128y - 5120 = 0$
(ii) The coordinates of P are $(-8, -64)$.		
Note that $\angle BGP = \angle CGP$ and GP is the perpendicular bisector of BC (the x-axis) .		
Let the coordinates of G be $(-8, t)$.	1M	for the x-coordinate of G or Q
$\because$ $GP = BG$		
$\therefore$ $(t-(-64))^2 = (-8-(-80))^2 + (t-0)^2$	1M	accept using $GP = CG$ or $BG = CG$
So, we have $t = \frac{17}{2}$.		
Since $GP = GQ$, the y-coordinate of Q is 81 .		
Thus, the coordinates of Q are $(-8, 81)$.	1A	pp–1 for missing '(' or ')'

Solution	Marks	Remarks
The coordinates of P are $(-8,-64)$.		
Note that $\angle BGP = \angle CGP$ and QP is the perpendicular bisector of BC (the x-axis) .		
The x-coordinate of Q is -8 .	1M	for the x-coordinate of Q or G
Let the coordinates of Q be $(-8,b)$.		
Note that $\angle PBQ = 90^\circ$.		
$\left(\frac{b-0}{-8-(-80)}\right)\left(\frac{-64-0}{-8-(-80)}\right)=-1$	1M	accept using similar triangles
$\left(\frac{b}{72}\right)\left(\frac{-64}{72}\right)=-1$		
$b=81$	1A	
Thus, the coordinates of Q are $(-8,81)$.		pp−1 for missing '(' or ')'
(iii) The slope of $BQ = \frac{81-0}{-8-(-80)} = \frac{9}{8}$		
The slope of $IQ = \frac{81-32}{-8-0} = \frac{-49}{8}$		
$\because$ (the slope of BQ)(the slope of IQ) $= \frac{-441}{64} \neq -1$	1M	for testing whether $BQ \perp IQ$
$\therefore \angle BQI \neq 90^\circ$		
Note that $\angle BRI = 90^\circ$.		
So, we have $\angle BQI + \angle BRI \neq 180^\circ$.		
Thus, B , Q , I and R are not concyclic.	1A	f.t.
For the equation of the circle which passes through B , I and R ,		
$\because \angle BRI = 90^\circ$		
$\therefore$ the centre of the circle is the mid-point of BI .		
Therefore, the coordinates of the centre are $(-40,16)$.		
The radius $= \sqrt{(-40-0)^2+(16-32)^2} = \sqrt{1856}$		
So, the equation of the circle which passes through B , I and R is $(x+40)^2+(y-16)^2=1856$.		
Note that $(-8+40)^2+(81-16)^2 = 5249 \neq 1856$.	1M	for testing whether the fourth point lies on the circle
So, Q does not lie on the circle which passes through B , I and R .		
Thus, B , R , I and Q are not concyclic.	1A	f.t.
Let the equation of the circle which passes through B , I and R be $x^2+y^2+lx+my+n=0$.		
Since the coordinates of R are $(0,0)$, we have $n=0$.		
$\because$ the coordinates of B and I are $(-80,0)$ and $(0,32)$ respectively.		
$\therefore$ we have $(-80)^2+l(-80)+m(0)=0$ and $(32)^2+m(32)=0$.		
Hence, we have $l=80$ and $m=-32$.		
So, the equation of the circle which passes through B , I and R is $x^2+y^2+80x-32y=0$.		
Note that $(-8)^2+81^2+80(-8)-32(81)=3393 \neq 0$.	1M	for testing whether the fourth point lies on the circle
So, Q does not lie on the circle which passes through B , I and R .		
Thus, B , R , I and Q are not concyclic.	1A	f.t.
	----------(8)	

2008 Mathematics

卷二
Paper 2

題號 Question No.	答案 Key	題號 Question No.	答案 Key
1.	B (39)	31.	D (40)
2.	A (87)	32.	D (40)
3.	A (64)	33.	B (51)
4.	B (85)	34.	D (64)
5.	C (49)	35.	A (36)
6.	C (74)	36.	A (88)
7.	C (84)	37.	D (59)
8.	D (69)	38.	C (60)
9.	A (25)	39.	A (82)
10.	A (55)	40.	A (49)
11.	C (77)	41.	B (69)
12.	A (45)	42.	B (40)
13.	C (57)	43.	B (59)
14.	D (53)	44.	B (45)
15.	C (68)	45.	B (40)
16.	B (52)	46.	B (39)
17.	C (51)	47.	B (44)
18.	C (60)	48.	D (42)
19.	C (82)	49.	D (39)
20.	C (62)	50.	A (37)
21.	D (45)	51.	B (44)
22.	C (62)	52.	D (26)
23.	A (66)	53.	D (39)
24.	A (76)	54.	A (55)
25.	A (82)		
26.	D (60)		
27.	B (69)		
28.	D (65)		
29.	B (79)		
30.	C (44)		

註： 括號內數字爲答對百分率。
Note: Figures in brackets indicate the percentages of candidates choosing the correct answers.

Candidates' Performance

Paper 1

Candidates generally performed better in Section A than in Section B. More candidates were willing to attempt those parts of the questions in Section B that fall within the Foundation Part of the Whole Syllabus.

Section A(1) (Compulsory)

Question Number	Performance in General
1	Good. Some candidates wrongly thought that $\frac{a^3b^3}{a^2}=\frac{b^3}{a^{3-2}}$ and hence gave $\frac{b^3}{a}$ as the answer. Quite a number of candidates wrote an '$=$' sign at the beginning of their working.
2 (a)	Good. Some candidates mistakenly simplified $\frac{14x}{5}\geq 2x+7$ as $14x\geq 10x+7$.
(b)	Quite good. Some candidates mistakenly gave 8 as the answer.
3 (a)	Very good. Only a few candidates could not write down the correct answer.
(b)	Fair. Some candidates gave the values of m instead of k .
4	Good. Some candidates wrongly gave W50°S or 040 as the answer.
5	Good. Some candidates did not simplify the final answer.
6 (a)	Very good. Most candidates could obtain the correct answer. However, a few candidates gave $s=\frac{2t}{5}$ as the answer.
(b)	Very good. Most candidates could use the result in (a) to do this part.
7 (a)	Poor. Most candidates did not know the method of 'rounding up'.
(b)	Poor. Since most candidates could not obtain the answer in (a) by using the correct method, they could not give an acceptable explanation in this part.
8 (a)	Very good. Most candidates could obtain the correct answer.
(b) (i)	Very good. Most candidates could obtain the correct answer.
(ii)	Fair. Only a few candidates could directly write down the correct answer. Some of them gave 20 as the answer.
9	Very good. Some candidates could not obtain the correct answer for z .

Section A(2) (Compulsory)

Question Number	Performance in General
10 (a)	Very good. Most candidates could obtain the answers. However, many candidates did not show the steps for finding b and c .
(b)	Good. Quite a number of candidates did not write down the unit in their answers.
11 (a)	Good. Many candidates could correctly find the value of b by substituting corresponding x and y in the given equation. Quite a number of candidates wrongly treated x-intercept as an ordered pair instead of a real number.
(b)	Fair. Many candidates wrongly used $f(x)=0$ instead of $f(x)=k$. Some candidates had difficulty in handling the inequality $\Delta>0$.
(c)	Fair. Only a few candidates could directly write down the correct answer.
12 (a)	Good. Many candidates could obtain the correct answer.
(b)	Good. Many candidates just found out the slopes of OB and OC and then gave conclusions without comparing the slopes explicitly.
(c)	Fair. Many candidates obtained a wrong slope for CD . Many candidates knew that the y-coordinate of D is 3 .
13 (a)	Good. Most candidates could obtain the correct answer but some candidates did not write down the unit.
(b)	Very good. Most candidates could use the result of (a) to obtain the correct answer.
(c)	Fair. Many candidates could not clearly explain whether X and Y were similar.

Section B (A choice of 3 out of 4 questions)

Question Number	Popularity %	Performance in General
14 (a)(i)	94	Good. Most candidates could obtain the correct answer but some of them did not simplify their answers.
(ii) (1)		Good. Most candidates could obtain the correct answer but some of them did not simplify their answers.
(2)		Very good. Many candidates could obtain the correct answer.
(b)(i)		Fair. Most candidates knew what median was but quite a number of them did not know what inter-quartile range was.
(ii)		Poor. Most candidates could not clearly describe how the manager should raise the suggested bonus.
15 (a)	92	Good. Many candidates could obtain the correct answer by using sine formula.
(b)(i)		Good. Many candidates could obtain the correct answer by using cosine formula.
(ii)		Poor. Many candidates did not define the required angle clearly.
(iii)		Fair. Many candidates knew the angle obtained in (b)(ii) is the inclination of the line of the greatest slope.
16 (a)	83	Good. Many candidates did not show the steps for finding b and c .
(b)(i)		Poor. Only a few candidates could obtain the correct answer.
(ii)		Poor. Many candidates tried to use the result in (b)(i) but most of them found difficulty in finding the salaries tax by using the given taxation rates.
(c)		Poor. Most candidates did not know how to find the amount of money saved. Some of them did not divide the interest rate by 12 , and some of them could not get the correct number of terms.
17 (a)	31	Poor. Most candidates who attempted this part could only prove that $BP = CP$.
(b)(i)		Good. Many candidates could find the equation of the circle.
(ii)		Fair. Many candidates knew that the x-coordinate of Q is -8 . However, only a few candidates attempted to find the y-coordinate of Q .
(iii)		Fair. Many candidates could not explain why the four points are not concyclic.

General recommendations

Candidates are advised to:

1. revise fundamental mathematics topics like percentages, factorization, estimation, ratio, percentage changes, congruency and similarity;

2. show all working;

3. define any symbols used;

4. write down the unit of the answer if necessary;

5. practice more on problems involving geometric proofs;

6. develop a better spatial sense, such as distinguishing right-angled triangles from non right-angled triangles in 3D diagrams;

7. make use of the memory space in calculators for carrying more significant figures throughout the working in solving trigonometric problems;

8. trace the co-relation between different parts of a question, particularly in the long questions;

9. present solutions clearly; and

10. simplify the answer if necessary.

Paper 2

The paper consisted of 54 multiple-choice items. Section A comprised 36 questions on the Foundation Part and Section B 18 questions on the Whole Syllabus. Post-examination analysis revealed the following:

1. Candidates' performance on Items 2, 4, 6, 7, 11, 19, 24, 25, 29, 36 and 39 was good. Over 70% of the candidates answered them correctly.

2. Candidates' performance on Items 9 and 52 was unsatisfactory. Less than 30% of the candidates gave the correct answer.

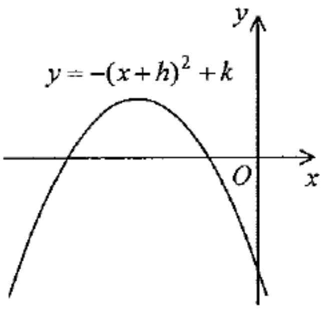
3. In Item 1, many candidates wrongly thought that $\left(\frac{1}{2}\right)^{888}$ was equal to zero, and hence wrongly gave Option C as the answer.

   Q.1 $\left(\frac{1}{2}\right)^{888}(-2)^{887} =$

	A.	-2 .	(7%)
*	B.	-0.5 .	(39%)
	C.	0 .	(48%)
	D.	0.5 .	(6%)

4. In Item 9, although many candidates knew that the y-coordinate of the vertex is k, they wrongly thought that the x-coordinate of the vertex was h. Hence they wrongly gave Option C as the answer.

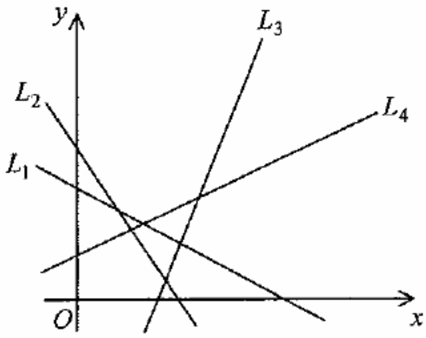
   Q.9 The figure shows the graph of $y = -(x+h)^2 + k$. Which of the following must be true?



*	A.	$h > 0$ and $k > 0$	(25%)
	B.	$h > 0$ and $k < 0$	(22%)
	C.	$h < 0$ and $k > 0$	(34%)
	D.	$h < 0$ and $k < 0$	(19%)

5. In Item 32, a number of candidates were not familiar with the comparison of the magnitudes of the negative slopes and hence wrongly gave Option B as the answer.

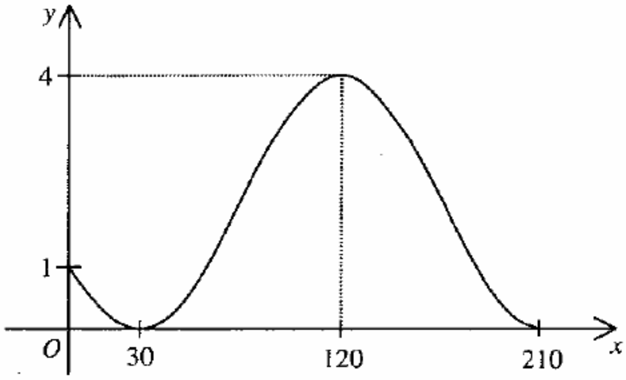
   Q.32 In the figure, L_1 , L_2 , L_3 and L_4 are straight lines. If m_1 , m_2 , m_3 and m_4 are the slopes of L_1 , L_2 , L_3 and L_4 respectively, which of the following must be true?



	A.	$m_1 < m_2 < m_3 < m_4$	(13%)
	B.	$m_1 < m_2 < m_4 < m_3$	(32%)
	C.	$m_2 < m_1 < m_3 < m_4$	(15%)
*	D.	$m_2 < m_1 < m_4 < m_3$	(40%)

6. In Item 46, many candidates were not familiar with the transformation of the trigonometric functions, and hence gave wrong answers.

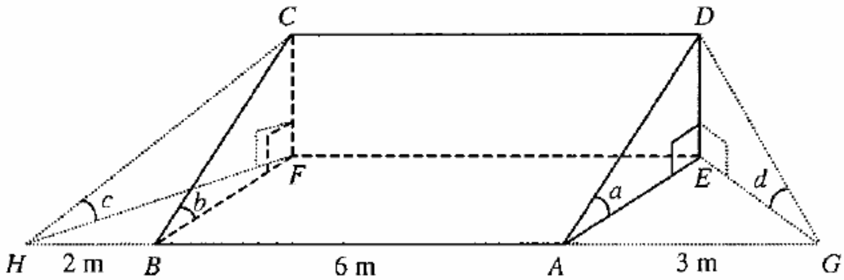
Q.46 Let a and b be constants. If the figure shows the graph of $y = a\cos(2x° + 120°) + b$, then



	A.	$a = 1$ and $b = 3$.	(20%)
*	B.	$a = 2$ and $b = 2$.	(39%)
	C.	$a = 3$ and $b = 1$.	(22%)
	D.	$a = 4$ and $b = 0$.	(19%)

7. In Item 49, many candidates mistakenly thought that c was the smallest one among the four angles, and hence wrongly gave Option B or C as the answer.

Q.49 The figure shows a right prism $ABCDEF$ with a right-angled triangle as the cross-section. A , B , E and F lie on the horizontal ground. G and H are two points on the horizontal ground so that G , A , B and H are collinear. It is given that $AB = 6$ m , $AG = 3$ m and $BH = 2$ m . If $\angle DAE = a$, $\angle CBF = b$, $\angle CHF = c$ and $\angle DGE = d$, which of the following must be true?



	A.	$a < d < c$	(11%)
	B.	$c < a < d$	(22%)
	C.	$c < d < b$	(28%)
*	D.	$d < c < b$	(39%)

8. In Item 52, many candidates were not familiar with the properties of the orthocentre of a triangle and hence gave wrong answers.

Q.52 Let O be the origin. If the coordinates of the points A and B are $(48, 0)$ and $(24, 18)$ respectively, then the y-coordinate of the orthocentre of ΔABO is

	A.	-7 .	(9%)
	B.	6 .	(27%)
	C.	8 .	(38%)
*	D.	32 .	(26%)