2008 Additional Mathematics

評卷參考*

* 此部分只設英文版本。

Marking Scheme

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

General Instructions To Markers

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.

5. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

6. In the marking scheme, marks are classified into the following three categories:

'M' marks	–	awarded for applying correct methods
'A' marks	–	awarded for the accuracy of the answers
Marks without 'M' or 'A'	–	awarded for correctly completing a proof or arriving at an answer given in the question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should NOT be awarded, unless otherwise specified.

7. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles, whereas alternative answers are enclosed by solid rectangles.

8. (a) Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.
   (b) In case a certain degree of accuracy had been specified in the question, answers not accurate up to that degree should not be accepted. For answers with an excess degree of accuracy, deduct 1 mark (*pp*) for the first time it happened. In any case, do not deduct any marks for excess degree of accuracy in those steps where candidates failed to score any marks.

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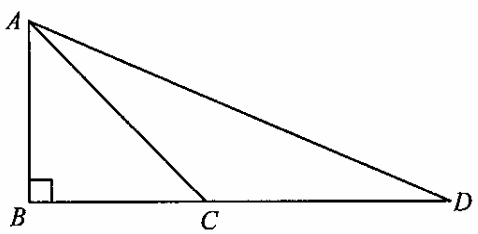
9. Marks may be deducted for poor presentation (*pp*). The symbol (*pp-1*) should be used to denote 1 mark deducted for *pp*.
   (a) In section A, at most deduct 1 mark for *pp* in each question, up to a maximum of 2 marks. For similar *pp*, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in section A for the same *pp*.
   (b) In section B, at most deduct 1 mark for *pp* in the whole section.
   (c) In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.

10. In section B, marks may be deducted for wrong / no unit (*u*). The symbol (*u-1*) should be used to denote 1 mark deducted for *u*.
   (a) At most deduct 1 mark for *u* in the whole section B.
   (b) In any case, do not deduct any marks for *u* in those steps where candidates could not score any marks.

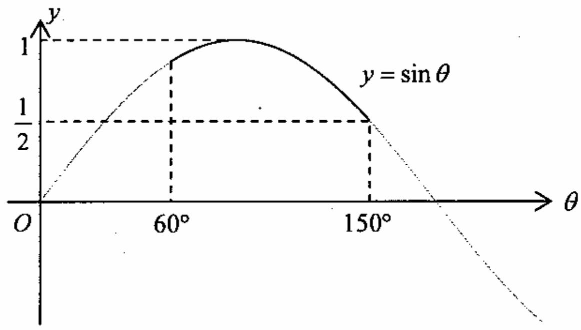
11. Marks entered in the Page Total Box should be the NET total scored on that page.

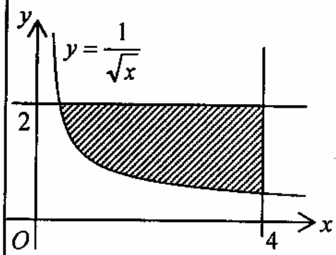
Solution	Marks	Remarks
1. $\int (8x+5)^{250} \mathrm{d}x$		
$= \dfrac{(8x+5)^{251}}{251\,(8)} + c$	1M	For $\dfrac{(8x+5)^{251}}{251}$
$= \dfrac{(8x+5)^{251}}{2008} + c$	1A	(pp-1) if c was omitted
	(2)	
2. (a) $\left(2x+\dfrac{1}{x}\right)^3 = (2x)^3 + 3(2x)^2\left(\dfrac{1}{x}\right) + 3(2x)\left(\dfrac{1}{x}\right)^2 + \left(\dfrac{1}{x}\right)^3$	1M	For binomial coef. (1,3,3,1) (Accept C_r^3 notation)
Alternative Solution $\left(2x+\dfrac{1}{x}\right)^3 = \left(2x+\dfrac{1}{x}\right)\left(4x^2+4+\dfrac{1}{x^2}\right)$	1A	For $4x^2+4+\dfrac{1}{x^2}$
$= 8x^3 + 12x + \dfrac{6}{x} + \dfrac{1}{x^3}$	1A	
(b) $(3x^2-x-5)\left(2x+\dfrac{1}{x}\right)^3 = (3x^2-x-5)\left(8x^3+12x+\dfrac{6}{x}+\dfrac{1}{x^3}\right)$		
$\therefore$ the coefficient of $x = 3\cdot 6 - 5\cdot 12$	1M	For collecting like terms
$= -42$	1A	
	(4)	
3. Let $t = \tan 22.5°$.		
$\because \tan(2\times 22.5°) = \dfrac{2\tan 22.5°}{1-\tan^2 22.5°}$	1M	For $45° = 2\times 22.5°$
$\therefore 1 = \dfrac{2t}{1-t^2}$	1A	OR $1 = \dfrac{2\tan 22.5°}{1-\tan^2 22.5°}$
$t^2 + 2t - 1 = 0$		
$t = \dfrac{-2+\sqrt{8}}{2}$ or $\dfrac{-2-\sqrt{8}}{2}$ (rejected since $t>0$)	1A	
i.e. $t = \sqrt{2}-1$	1A	
Alternative Solution (1) Let $t = \tan 22.5°$. $\tan 22.5° = \tan(45° - 22.5°)$	1M	For $22.5° = 45° - 22.5°$
$= \dfrac{\tan 45° - \tan 22.5°}{1+\tan 45° \tan 22.5°}$		
i.e. $t = \dfrac{1-t}{1+t}$	1A	OR $\tan 22.5° = \dfrac{1-\tan 22.5°}{1+\tan 22.5°}$
$t^2 + 2t - 1 = 0$		
$t = \dfrac{-2+\sqrt{8}}{2}$ or $\dfrac{-2-\sqrt{8}}{2}$ (rejected since $t>0$)	1A	
i.e. $t = \sqrt{2}-1$	1A	

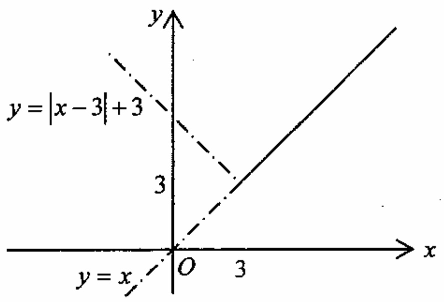
Solution	Marks	Remarks
<u>Alternative Solution (2)</u>		
$\tan 22.5° = \dfrac{\sin 22.5°}{\cos 22.5°}$		
$= \dfrac{2\sin 22.5° \cos 22.5°}{2\cos^2 22.5°}$		
$= \dfrac{\sin 45°}{1+\cos 45°}$	1M+1A	1M for $45° = 2 \times 22.5°$
$= \dfrac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$	1A	
$= \dfrac{1}{\sqrt{2}+1}$	1A	
$= \sqrt{2}-1$		
<u>Alternative Solution (3)</u>		
$\tan 22.5° = \dfrac{\sin 22.5°}{\cos 22.5°}$		
$= \dfrac{\sqrt{\sin^2 22.5°}}{\sqrt{\cos^2 22.5°}}$ (since $\sin 22.5°$ and $\cos 22.5°$ are both positive)		
$= \sqrt{\dfrac{\frac{1}{2}(1-\cos 45°)}{\frac{1}{2}(1+\cos 45°)}}$	1M+1A	1M for $45° = 2 \times 22.5°$
$= \sqrt{\dfrac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}}$	1A	
$= \sqrt{\dfrac{\sqrt{2}-1}{\sqrt{2}+1}}$		
$= \sqrt{\left(\sqrt{2}-1\right)^2}$		
$= \sqrt{2}-1$	1A	
<u>Alternative Solution (4)</u>		
Construct a right-angled triangle ABC with $AB = BC = 1$ and extend BC to D such that $AC = CD$.	1M	
		
$\angle ACB = 45°$		
$\angle ADC = \dfrac{\angle ACB}{2} = 22.5°$	1A	
$AC = \sqrt{1^2+1^2} = \sqrt{2}$		
$\therefore CD = \sqrt{2}$	1A	

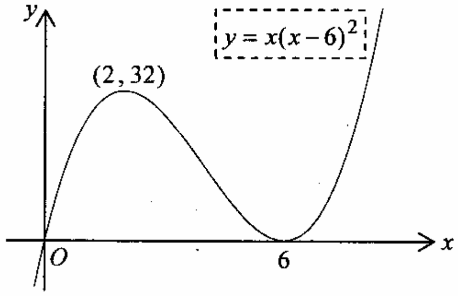
Solution	Marks	Remarks
$\because \tan\angle ADB = \frac{AB}{BD}$ $\therefore \tan 22.5° = \frac{1}{1+\sqrt{2}}$ $= \sqrt{2}-1$	1A	
	(4)	
4. Since the graph of $y = kx^2 - x + 9k$ lies below the x-axis, $(-1)^2 - 4(k)(9k) < 0$ and $k < 0$	1M+1A	1M for $\Delta < 0$ 1A for $k < 0$
Alternative Solution $kx^2 - x + 9k = k\left[x^2 - \frac{1}{k}x + \frac{1}{(2k)^2} - \frac{1}{4k^2}\right] + 9k$	1M	1M for completing square
$= k\left(x - \frac{1}{2k}\right)^2 + 9k - \frac{1}{4k}$		
Since the graph of $y = kx^2 - x + 9k$ lies below the x-axis, $9k - \frac{1}{4k} < 0$ and $k < 0$	1A	
$k^2 > \frac{1}{36}$ and $k < 0$		
$\left(k > \frac{1}{6} \text{ or } k < \frac{-1}{6}\right)$ and $k < 0$	1M	For solving a quad ineq correctly
$k < \frac{-1}{6}$	1A	
	(4)	
5. For $n = 1$, L.H.S. $= 1^3 = 1$ R.H.S. $= \frac{1}{4}(1)^2(1+1)^2 = 1$		
$\therefore$ L.H.S. = R.H.S. and so the statement is true for $n = 1$.	1	(pp-1) if "the statement is true for $n = 1$ " was omitted
Assume $1^3 + 2^3 + 3^3 + \cdots + k^3 = \frac{1}{4}k^2(k+1)^2$, where k is a positive integer.	1	
$\therefore 1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$ (by the assumption)	1	
$= \frac{1}{4}(k+1)^2\left[k^2 + 4(k+1)\right]$		
$= \frac{1}{4}(k+1)^2(k+2)^2$	1	
$= \frac{1}{4}(k+1)^2\left[(k+1)+1\right]^2$		
Hence the statement is also true for $n = k+1$.		(pp-1) if this line was omitted
By the principle of mathematical induction, the statement is true for all positive integers n.	1	Follow through
	(5)	

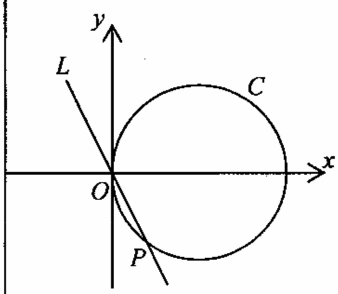
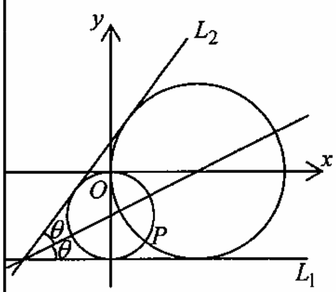
Solution	Marks	Remarks
6. $y=\dfrac{3x}{x^2+2}$		
$\dfrac{dy}{dx}=\dfrac{(x^2+2)(3)-(3x)(2x)}{(x^2+2)^2}$	1M+1A	OR $3(x^2+2)^{-1}+3x[-(x^2+2)^{-2}\cdot 2x]$ 1M for using differentiation
$=\dfrac{6-3x^2}{(x^2+2)^2}$		
$\therefore \left.\dfrac{dy}{dx}\right\vert_{x=2}=\dfrac{-1}{6}$	1A	
Hence the equation of the tangent at $(2,1)$ is		
$y-1=\dfrac{-1}{6}(x-2)$	1M	For pt-slope form of tangent
Alternative Solution Let the equation of the tangent be $y=\dfrac{-1}{6}x+c$ $\therefore 1=\dfrac{-1}{6}(2)+c$ $c=\dfrac{4}{3}$	1M	
i.e. the equation of tangent is $x+6y-8=0$	1A	
	(5)	
7. $\because \overrightarrow{PB}=2\overrightarrow{AP}$		
$\therefore \overrightarrow{OP}=\dfrac{2\,\overrightarrow{OA}+\overrightarrow{OB}}{3}$	1M	For $\dfrac{2\mathbf{u}+\mathbf{v}}{3}$
$=\dfrac{2(2\mathbf{i}+3\mathbf{j})+(5\mathbf{i}+6\mathbf{j})}{3}$		
$=3\mathbf{i}+4\mathbf{j}$	1A	
Alternative Solution $\because \overrightarrow{PB}=2\overrightarrow{AP}$ $\therefore \overrightarrow{OB}-\overrightarrow{OP}=2(\overrightarrow{OP}-\overrightarrow{OA})$ $5\mathbf{i}+6\mathbf{j}-\overrightarrow{OP}=2\,\overrightarrow{OP}-2(2\mathbf{i}+3\mathbf{j})$ $\overrightarrow{OP}=3\mathbf{i}+4\mathbf{j}$	1M 1A	
$\lvert\overrightarrow{OP}\rvert=\sqrt{3^2+4^2}$	1M	
$=5$		
Hence the unit vector in the direction of $\overrightarrow{OP}=\dfrac{3\mathbf{i}+4\mathbf{j}}{5}$.	1M+1A	(pp-1) for omitting vector signs in most cases
	(5)	
8. Let P be (x,y) .		
$2x+y-3=0$ -- (1)		
$\left\vert \dfrac{1}{2}\begin{vmatrix}1&3\\2&2\\x&y\\1&3\end{vmatrix}\right\vert=1$	1M	For $\dfrac{1}{2}\begin{vmatrix}1&3\\2&2\\x&y\\1&3\end{vmatrix}$

Solution	Marks	Remarks
$2+2y+3x-6-2x-y=\pm 2$	1M	For expansion and $\pm$ sign
$x+y=6$ or $x+y=2$ -- (2)	1A	For either one
Alternative Solution		
Since P lies on $2x+y-3=0$, therefore we can let P be $(x,3-2x)$.	1A	OR $\left(\frac{3-y}{2},y\right)$
$\left\lvert \frac{1}{2}\begin{vmatrix}1 & 3\\ 2 & 2\\ x & 3-2x\\ 1 & 3\end{vmatrix}\right\rvert=1$	1M	For $\frac{1}{2}\begin{vmatrix}1 & 3\\ 2 & 2\\ x & 3-2x\\ 1 & 3\end{vmatrix}$
$2+6-4x+3x-6-2x-3+2x=\pm 2$	1M	For expansion and $\pm$ sign
Solving for x : $x=-3$ or 1	1A	OR $y=9$ or 1
$\therefore (x,y)=(-3,9)$ or $(1,1)$ (rejected since P lies on the 2nd quadrant)		
i.e. $(x,y)=(-3,9)$	1A	
	(5)	
9. (a) Let $\sin x+\sqrt{3}\cos x=r\sin(x+\alpha)$		
$=r\sin x\cos\alpha+r\cos x\sin\alpha$		
$\therefore r\cos\alpha=1$ and $r\sin\alpha=\sqrt{3}$		For either r or α
Solving, $r=2$ and $\alpha=60°$.	1A	(pp-1) for $\alpha=\frac{\pi}{3}$
i.e. $\sin x+\sqrt{3}\cos x=2\sin(x+60°)$	1A	(pp-1) if this line was omitted
(b) $\because 0°\le x\le 90°$		
$\therefore 60°\le x+60°\le 150°$		
		
Reading from the graph, $\frac{1}{2}\le\sin(x+60°)\le 1$	1M	
$\therefore 1\le 2\sin(x+60°)\le 2$		
Alternative Solution		
For $x=0°$, $\sin(x+60°)=\frac{\sqrt{3}}{2}$		
$y=\sin(x+60°)$ is increasing for $0°\le x\le 30°$		
For $x=30°$, $\sin(x+60°)=1$ which is the greatest value	1M	
$y=\sin(x+60°)$ is decreasing for $30°\le x\le 90°$		
For $x=90°$, $\sin(x+60°)=\frac{1}{2}$		
Hence the least value of $\sin(x+60°)=\frac{1}{2}$ for $0°\le x\le 90°$.		
Therefore the least value of $\sin x+\sqrt{3}\cos x$ is 1 ,	1A	
and the greatest value of $\sin x+\sqrt{3}\cos x$ is 2 .	1A	
	(5)	

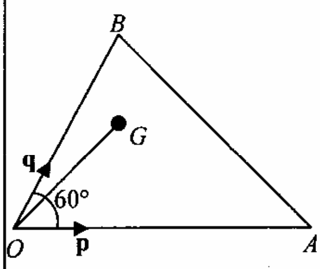
Solution	Marks	Remarks
10. Solving for x from $y=\frac{1}{\sqrt{x}}$ and $y=2$, we have $x=\frac{1}{4}$.	1A	
Area $=\int_{\frac{1}{4}}^{4}\left(2-\frac{1}{\sqrt{x}}\right)dx$	1M+1A	1M for $\int(y_1-y_2)\,dx$
$=\left[2x-2\sqrt{x}\right]_{\frac{1}{4}}^{4}$	1A	For $2x-2\sqrt{x}$
$=\frac{9}{2}$	1A	
Alternative Solution (1)		
Solving for x from $y=\frac{1}{\sqrt{x}}$ and $y=2$, we have $x=\frac{1}{4}$.	1A	
Area $=2\left(4-\frac{1}{4}\right)-\int_{\frac{1}{4}}^{4}\frac{1}{\sqrt{x}}dx$	1M+1A	1M for "rectangle $-\int y\,dx$"
$=\frac{15}{2}-\left[2\sqrt{x}\right]_{\frac{1}{4}}^{4}$	1A	For $2\sqrt{x}$
$=\frac{9}{2}$	1A	
Alternative Solution (2)		
Solving for y from $y=\frac{1}{\sqrt{x}}$ and $x=4$, we have $y=\frac{1}{2}$.	1A	
Area $=\int_{\frac{1}{2}}^{2}\left(4-\frac{1}{y^2}\right)dy$	1M+1A	1M for $\int(x_1-x_2)\,dy$
$=\left[4y+\frac{1}{y}\right]_{\frac{1}{2}}^{2}$	1A	For $4y+\frac{1}{y}$
$=\frac{9}{2}$	1A	
	(5)	
11. (a) For $x\le 3$, $\lvert x-3\rvert+3=x$ becomes		
$-(x-3)+3=x$	1A	For $\lvert x-3\rvert=-(x-3)$
$x=3$	1A	
Alternative Solution		
$\lvert x-3\rvert+3=x$		
$\lvert x-3\rvert=x-3$		
$\therefore x-3\ge 0$	1A	
$x\ge 3$		
$\because x\le 3$, $\therefore$ the solution can only be $x=3$	1A	
(b) For $x>3$, $\lvert x-3\rvert+3=x$ becomes	1M	For considering $x>3$
$x-3+3=x$		
which is true for all real x		
Hence the solution in this case is $x>3$.	1A	
Hence the overall solution is $x\ge 3$.	1A	

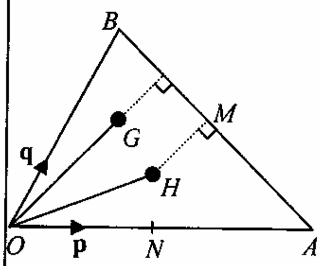
Solution	Marks	Remarks
<u>Alternative Solution (1)</u> $\lvert x-3\rvert+3=x$ $\lvert x-3\rvert=x-3$ $\therefore x-3\geq 0$ $x\geq 3$	 1M+1A 1A	 1M for using def. of $\lvert f(x)\rvert$
<u>Alternative Solution (2)</u>  From the graph, we see that the condition for $\lvert x-3\rvert+3=x$ is $x\geq 3$.	1M 1A 1A	For attempting to use graphical method (Drawing at least 1 line) For either graph
	(5)	
12. (a) The equation of L_2 is $x+ky=0$ Solving L_2 and $L_1: y=kx+1$, we have $x=\dfrac{-k}{k^2+1}$ ---------- (1) $\therefore y=\dfrac{1}{k^2+1}$ ---------- (2) i.e. the coordinates of P are $\left(\dfrac{-k}{k^2+1},\dfrac{1}{k^2+1}\right)$	1A 1A 1A	
(b) (1) ÷ (2) : $k=\dfrac{-x}{y}$ ---------- (3) Substitute (3) into (2) : $y=\dfrac{1}{\left(\dfrac{-x}{y}\right)^2+1}$ $\therefore 1=\dfrac{y}{x^2+y^2}$	1M 1M 1A	For writing k as the subject For eliminating k
<u>Alternative Solution</u> By (2), $k=\pm\sqrt{\dfrac{1}{y}-1}$ ---------- (3) Substitute (3) into (2) : $x=\dfrac{\mp\sqrt{\dfrac{1}{y}-1}}{\dfrac{1}{y}-1+1}$ $\therefore x^2=y^2\left(\dfrac{1}{y}-1\right)$	1M 1M 1A	Accept without ± sign
i.e. $x^2+y^2-y=0$ (excluding $(0,0)$) which is the locus of P .		
	(6)	

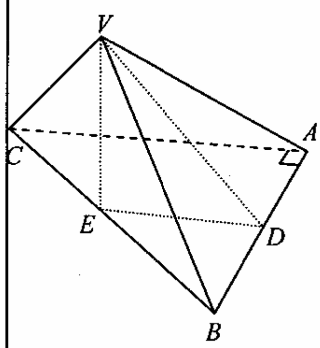
Solution	Marks	Remarks
13. (a) $y = x(x-6)^2$		
$\frac{dy}{dx} = (x-6)^2 + x \cdot 2(x-6)$	1M	For product rule
$= 3x^2 - 24x + 36$		
Alternative Solution $y = x^3 - 12x^2 + 36x$		
$\frac{dy}{dx} = 3x^2 - 24x + 36$	1M	
$\frac{dy}{dx} = 0$		
$3x^2 - 24x + 36 = 0$	1M	
$x = 2$ or 6		
$\frac{d^2y}{dx^2} = 6x - 24$		
$\left.\frac{d^2y}{dx^2}\right\|_{x=2}$ $(= -12) < 0$ and $\left.\frac{d^2y}{dx^2}\right\|_{x=6}$ $(= 12) > 0$	1M	OR using sign test
$\therefore$ maximum point is $(2, 32)$ and minimum point is $(6, 0)$.	1A+1A	
(b) 		
	1M	For shape
	1A	For all correct (pp-1) for axes or origin not labelled, or origin / arrow sign missed
	(7)	

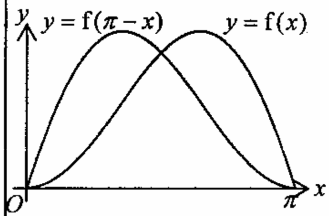
Solution	Marks	Remarks
14. (a) All circles in F pass through the intersections of $\begin{cases} x^2+y^2-10x=0 \\ 2x+y=0 \end{cases}$	1A	
Alternative Solution Substitute, say, $k=0$ into F: $x^2+y^2-10x=0$ Substitute, say, $k=1$ into F: $x^2+y^2-8x+y=0$	} 1A	
Solving, we get $\begin{cases} x=0 \\ y=0 \end{cases}$ or $\begin{cases} x=2 \\ y=-4 \end{cases}$		
$\therefore$ the coordinates of P are $(2,-4)$.	1A	
	(2)	
(b) $F: x^2+y^2+(2k-10)x+ky=0$	1A	
Centre of $F=\left(-k+5,\dfrac{-k}{2}\right)$	1A	
Let $x=-k+5$ and $y=\dfrac{-k}{2}$		
Eliminating k: $x=2y+5$ which is the required locus.	1M+1A	1M for elimination
Alternative Solution (1) The centres of circles in F lie on the straight line passing through the centre of $C: x^2+y^2-10x=0$ and perpendicular to $L: 2x+y=0$.		
Centre of $C=(5,0)$	1A	
Slope of $L=-2$	1A	
Hence the required locus is $y-0=\dfrac{1}{2}(x-5)$	1M	
i.e. $x-2y-5=0$	1A	
Alternative Solution (2) Since OP is a common chord to all circles in F, their centres lie on the perpendicular bisector of OP .		
Mid-point of $OP=(1,-2)$	1A	
Slope of $OP=-2$	1A	
Hence the required locus is $y+2=\dfrac{1}{2}(x-1)$	1M	
i.e. $x-2y-5=0$	1A	
	(4)	
(c) (i) By symmetry, L_1, L_2 and the straight line joining the two centres are concurrent.		
$\therefore Q$ is the intersection of $x-2y-5=0$ and $y+5=0$	1M	
Solving, $Q=(-5,-5)$	1A	
(ii) $x-2y-5=0$, whose slope is $\dfrac{1}{2}$, is the angle bisector of L_1 and L_2 .		
Let m be the slope of L_2 .		
$\therefore \left\lvert \dfrac{m-\frac{1}{2}}{1+\left(\frac{1}{2}\right)(m)} \right\rvert = \dfrac{1}{2}$	1M+1M	1M for $\dfrac{m_1-m_2}{1+m_1m_2}$ 1M for equating the slopes (Accept no absolute sign)
$m=\dfrac{4}{3}$ [or 0]	1A	

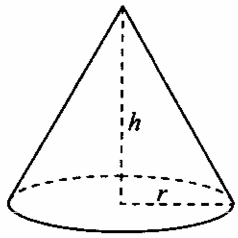
Solution	Marks	Remarks
Alternative Solution (1)		
Let the slope of $x-2y-5=0$ be $\tan\theta=\frac{1}{2}$.		
Therefore the slope of $L_2=\tan 2\theta$	1M	1M for slope of $L_2=\tan 2\theta$
$=\dfrac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2}=\frac{4}{3}$	1M+1A	1M for $\tan 2\theta=\dfrac{2\tan\theta}{1-\tan^2\theta}$
Alternative Solution (2)		
$\begin{cases}F: x^2+y^2+(2k-10)x+ky=0\\ L_1: y+5=0\end{cases}$		
$\therefore x^2+(-5)^2+(2k-10)x+k(-5)=0$		
$x^2+2(k-5)x+25-5k=0$		
For a circle in F touches L_1,		
$\Delta=2^2(k-5)^2-4(25-5k)=0$	1M	OR use method of "$d=r$"
$k^2-5k=0$		
$k=0$ or 5		
For $k=0$, the circle is $x^2+y^2-10x=0$ with centre $(5,0)$ and $r=5$	1A	For the eq of circle with either $k=0$ or 5, which is $x^2+y^2+5y=0$
By (i), let L_2 be $y+5=m(x+5)$		
i.e. $mx-y+5m-5=0$		
$\left\lvert\dfrac{m(5)-0+5m-5}{\sqrt{m^2+(-1)^2}}\right\rvert=5$	1M	OR use method of "$\Delta=0$"
$3m^2-4m=0$		
$m=\frac{4}{3}$ or 0 (rej)		
Hence the equation of L_2 is $y+5=\frac{4}{3}(x+5)$		
i.e. $4x-3y+5=0$	1A	
Alternative Solution of (c) (i) and (ii)		
$\begin{cases}F: x^2+y^2+(2k-10)x+ky=0\\ L_1: y+5=0\end{cases}$		
$\therefore x^2+(-5)^2+(2k-10)x+k(-5)=0$		
$x^2+2(k-5)x+25-5k=0$		
For a circle in F touches L_1,		
$\Delta=2^2(k-5)^2-4(25-5k)=0$	1M	OR use method of "$d=r$"
$k^2-5k=0$		
$k=0$ or 5		
For $k=0$, the circle is $x^2+y^2-10x=0$ with centre $(5,0)$ and $r=5$ For $k=5$, the circle is $x^2+y^2+5y=0$ with centre $\left(0,\frac{-5}{2}\right)$ and $r=\frac{5}{2}$	1A	For the eq of BOTH circles

Solution	Marks	Remarks
Let L_2 be $mx-y+c=0$.		
If L_2 is a common tangent to the two circles, then		
$\left\lvert\dfrac{m(5)-0+c}{\sqrt{m^2+(-1)^2}}\right\rvert=5$ and $\left\lvert\dfrac{0-\left(\frac{-5}{2}\right)+c}{\sqrt{m^2+(-1)^2}}\right\rvert=\dfrac{5}{2}$	1M	OR use method of " $\Delta=0$ "
$\therefore \begin{cases} 10mc=25-c^2 \\ 25m^2=4c^2+20c \end{cases}$		
Solving, $25\left(\dfrac{25-c^2}{10c}\right)^2=4c^2+20c$	1M	For eliminating m
$(5-c)^2(5+c)^2=4c^2\cdot 4c(c+5)$		
$(c+5)(15c^3+5c^2+25c-125)=0$		
$(c+5)(3c-5)(c^2+2c+5)=0$		
$c=\dfrac{5}{3}$ or -5		
$\therefore 10m\left(\dfrac{5}{3}\right)=25-\left(\dfrac{5}{3}\right)^2$ or $10m(-5)=25-(-5)^2$		
i.e. $m=\dfrac{4}{3}$ or 0		
Hence the equation of L_2 is $y=\dfrac{4}{3}x+\dfrac{5}{3}$	1A	
Therefore the intersection of L_1 and L_2 is $Q=(-5,-5)$.	1A	
	(6)	
15. (a) $\mathbf{p}\cdot\mathbf{q}=(1)(1)\cos 60^\circ$		
$=\dfrac{1}{2}$	1A	
	(1)	
(b) $\overrightarrow{OG}\cdot\overrightarrow{AB}=\left(\dfrac{2}{3}\mathbf{p}+\dfrac{5}{3}\mathbf{q}\right)\cdot(3\mathbf{q}-4\mathbf{p})$	1M	For considering $\overrightarrow{OG}\cdot\overrightarrow{AB}$
$=5\lvert\mathbf{q}\rvert^2+2\mathbf{p}\cdot\mathbf{q}-\dfrac{20}{3}\mathbf{p}\cdot\mathbf{q}-\dfrac{8}{3}\lvert\mathbf{p}\rvert^2$		
$=5(1)^2-\dfrac{14}{3}\left(\dfrac{1}{2}\right)-\dfrac{8}{3}(1)^2$	1M	
$=0$		
$\therefore OG\perp AB$	1	
$\overrightarrow{BG}\cdot\overrightarrow{OA}=\left(\dfrac{2}{3}\mathbf{p}+\dfrac{5}{3}\mathbf{q}-3\mathbf{q}\right)\cdot(4\mathbf{p})$	1A	For $\overrightarrow{BG}=\dfrac{2}{3}\mathbf{p}+\dfrac{5}{3}\mathbf{q}-3\mathbf{q}$
$=\dfrac{8}{3}\lvert\mathbf{p}\rvert^2-\dfrac{16}{3}\mathbf{p}\cdot\mathbf{q}$		
$=\dfrac{8}{3}(1)^2-\dfrac{16}{3}\left(\dfrac{1}{2}\right)$		Either one
$=0$		
$\overrightarrow{AG}\cdot\overrightarrow{OB}=\left(\dfrac{2}{3}\mathbf{p}+\dfrac{5}{3}\mathbf{q}-4\mathbf{p}\right)\cdot(3\mathbf{q})$		For $\overrightarrow{AG}=\dfrac{2}{3}\mathbf{p}+\dfrac{5}{3}\mathbf{q}-4\mathbf{p}$

Solution	Marks	Remarks
$= 5\lvert\mathbf{q}\rvert^2 - 10\mathbf{p}\cdot\mathbf{q}$		
$= 5(1)^2 - 10\left(\frac{1}{2}\right)$		
$= 0$		
Therefore G is the orthocentre of ΔOAB .	1	Follow through
	(5)	
(c) $\overrightarrow{HM} = t\left(\frac{2}{3}\mathbf{p} + \frac{5}{3}\mathbf{q}\right)$	1A	
$\overrightarrow{MN} = \frac{1}{2}\overrightarrow{BO} = \frac{-3\mathbf{q}}{2}$ (mid-point theorem)		
$\overrightarrow{HN} = \overrightarrow{HM} + \overrightarrow{MN}$	1M	OR $\overrightarrow{HM} + \overrightarrow{MO} + \overrightarrow{ON}$ OR $\overrightarrow{HM} + \overrightarrow{MA} + \overrightarrow{AN}$ OR $\overrightarrow{HM} + \overrightarrow{MB} + \overrightarrow{BO} + \overrightarrow{ON}$
$= \frac{2t}{3}\mathbf{p} + \left(\frac{5t}{3} - \frac{3}{2}\right)\mathbf{q}$	1A	
$\overrightarrow{HN}\cdot\overrightarrow{OA} = 0$		
$\left[\frac{2t}{3}\mathbf{p} + \left(\frac{5t}{3} - \frac{3}{2}\right)\mathbf{q}\right]\cdot(4\mathbf{p}) = 0$	1M	
$\frac{2t}{3}(1)^2 + \left(\frac{5t}{3} - \frac{3}{2}\right)\left(\frac{1}{2}\right) = 0$		
Alternative Solution $\overrightarrow{HN} \,//\, \overrightarrow{BG}$ $\left[\frac{2t}{3}\mathbf{p} + \left(\frac{5t}{3} - \frac{3}{2}\right)\mathbf{q}\right] // \left(\frac{2}{3}\mathbf{p} - \frac{4}{3}\mathbf{q}\right)$ $\dfrac{\frac{2t}{3}}{\frac{2}{3}} = \dfrac{\frac{5t}{3} - \frac{3}{2}}{\frac{-4}{3}}$	1M	
$t = \frac{1}{2}$	1A	
$\therefore \overrightarrow{OH} = \overrightarrow{OM} + \overrightarrow{MH}$		
$= \frac{4\mathbf{p} + 3\mathbf{q}}{2} - \left(\frac{1}{2}\right)\left(\frac{2}{3}\mathbf{p} + \frac{5}{3}\mathbf{q}\right)$		
$= \frac{5}{3}\mathbf{p} + \frac{2}{3}\mathbf{q}$	1A	(pp-1) for omitting vector / dot signs in most cases or using the expression $\mathbf{v}^2, \vec{v}^2$
Alternative Solution $\therefore \overrightarrow{OH} = \overrightarrow{ON} + \overrightarrow{NH}$ $= 2\mathbf{p} - \frac{2}{3}\left(\frac{1}{2}\right)\mathbf{p} - \left[\frac{5}{3}\left(\frac{1}{2}\right) - \frac{3}{2}\right]\mathbf{q}$ $= \frac{5}{3}\mathbf{p} + \frac{2}{3}\mathbf{q}$	1A	
	(6)	

Solution	Marks	Remarks
16. (a) $\because \angle VAB = 60^\circ \neq 90^\circ$,		
$\therefore$ the angle between the planes VAB and ABC cannot be represented by $\angle VAC$.	1	Follow through
	(1)	
(b) (i) $\because \Delta VAB$ is equilateral and D is the mid-point of AB		
$\therefore \angle VDB = 90^\circ$	1A	OR $VD \perp AB$
$\because D$ and E are mid-points of AB and BC respectively,		
$\therefore DE \,//\, AC$ (mid-point theorem)	1M	
$\therefore \angle BDE = \angle BAC = 90^\circ$ (corresponding angles, $DE \,//\, AC$)		
Hence the angle between the planes VAB and ABC **can be represented by** $\angle VDE$.	1	Follow through
(ii) It is given that $VA = AB = VB = VC = AC = 2$ cm		
$\because D$ and E are mid-points of AB **and** BC **respectively,**		
$\therefore DE = \frac{1}{2}AC = 1$ cm **(mid-point theorem)**	1A	
$BE = \sqrt{1^2 + 1^2} = \sqrt{2}$ **(Pythagoras' theorem)**		
$\because \Delta VBC$ is isosceles and E is the mid-point of BC ,		
$\therefore \angle VEB = 90^\circ$		
$\therefore VE = \sqrt{2^2 - \sqrt{2}^2} = \sqrt{2}$ cm	1A	
$VD = \sqrt{2^2 - 1^2} = \sqrt{3}$ cm (Pythagoras' theorem)	1A	
$\because VE^2 + ED^2 = \sqrt{2}^2 + 1^2 = 3 = VD^2$		
$\therefore \angle VED = 90^\circ$ (converse of Pythagoras' theorem)	1	Follow through
	(7)	
(c) Area of $\Delta ABC = \frac{1}{2}(2)(2) = 2\text{ cm}^2$		
$\because \angle VEB = \angle VED = 90^\circ$		
$\therefore VE$ is the height of the pyramid with respect to the base ABC .	1M	
Hence the volume of the pyramid		
$= \frac{1}{3}(2)\left(\sqrt{2}\right) = \frac{2\sqrt{2}}{3}\text{ cm}^3$	1A	
Area of $\Delta VAB = \frac{1}{2}(2)(\sqrt{3}) = \sqrt{3}\text{ cm}^2$		
Let h be the distance between C and plane VAB .		
$\therefore \frac{1}{3}\left(\sqrt{3}\right)h = \frac{2\sqrt{2}}{3}$	1M	
i.e. $h = 2\sqrt{\frac{2}{3}}$ cm	1A	(u-1) if unit was omitted
	(4)	

Solution	Marks	Remarks
17. (a) (i) $\int_0^{\pi}[\mathrm{f}(x)+\mathrm{f}(\pi-x)]\mathrm{d}x=\int_0^{\pi}[x\sin x+(\pi-x)\sin(\pi-x)]\mathrm{d}x$		
$=\int_0^{\pi}[x\sin x+(\pi-x)\sin x]\mathrm{d}x$	1A	For $(\pi-x)\sin x$
$=\pi\int_0^{\pi}\sin x\,\mathrm{d}x$	1A	For $\pi\sin x$
$=\pi[-\cos x]_0^{\pi}$	1A	For $-\cos x$
$=2\pi$	1	
(ii) By considering the symmetry of the graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{f}(\pi-x)$,		
$\int_0^{\pi}\mathrm{f}(x)\,\mathrm{d}x=\int_0^{\pi}\mathrm{f}(\pi-x)\,\mathrm{d}x$		
$\therefore\ \int_0^{\pi}\mathrm{f}(x)\,\mathrm{d}x=\frac{1}{2}\int_0^{\pi}[\mathrm{f}(x)+\mathrm{f}(\pi-x)]\mathrm{d}x$		
i.e. $\int_0^{\pi}x\sin x\,\mathrm{d}x=\pi$	1A	
	(5)	
(b) (i) $\frac{\mathrm{d}}{\mathrm{d}x}(x^2\sin x)=x^2\cos x+2x\sin x$	1A	
$\therefore\ \int_0^{\pi}x^2\cos x\,\mathrm{d}x=[x^2\sin x]_0^{\pi}-2\int_0^{\pi}x\sin x\,\mathrm{d}x$	1M	
$=-2\pi$ (by (a)(ii))	1A	
(ii) $\because\ \cos x=1-2\sin^2\frac{x}{2}$		
$\therefore\ \sin^2\frac{x}{2}=\frac{1-\cos x}{2}$	1A	
Hence the volume of revolution		
$=\pi\int_0^{\pi}(x\sin\frac{x}{2})^2\,\mathrm{d}x$	1M	For $\pi\int y^2\mathrm{d}x$
$=\pi\int_0^{\pi}x^2\sin^2\frac{x}{2}\mathrm{d}x$		
$=\pi\int_0^{\pi}x^2\left(\frac{1-\cos x}{2}\right)\mathrm{d}x$		
$=\frac{\pi}{2}\int_0^{\pi}x^2\,\mathrm{d}x-\frac{\pi}{2}\int_0^{\pi}x^2\cos x\,\mathrm{d}x$		
$=\frac{\pi}{2}\left[\frac{x^3}{3}\right]_0^{\pi}-\frac{\pi}{2}(-2\pi)$	1M	For using (b)(i)
$=\frac{\pi^4}{6}+\pi^2$	1A	
	(7)	

Solution	Marks	Remarks
18. (a) The volume of the cone is $V=\frac{1}{3}\pi r^2 h$.		
$\therefore 0=\frac{\pi}{3}\left(2r\frac{\mathrm{d}r}{\mathrm{d}t}h+r^2\frac{\mathrm{d}h}{\mathrm{d}t}\right)$	1M+1A	1M for product rule (Accept diff. wrt r or h)
$2\frac{\mathrm{d}r}{\mathrm{d}t}h+r(-2)=0$	1A	For $\frac{\mathrm{d}h}{\mathrm{d}t}=-2$
$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{r}{h}$	1	
	(4)	
(b) (i) $S=\pi r\sqrt{h^2+r^2}$	1A	
$S^2=\pi^2(r^2h^2+r^4)$		
$\frac{\mathrm{d}}{\mathrm{d}t}(S^2)=\pi^2\left(r^2\cdot 2h\frac{\mathrm{d}h}{\mathrm{d}t}+2r\frac{\mathrm{d}r}{\mathrm{d}t}\cdot h^2+4r^3\frac{\mathrm{d}r}{\mathrm{d}t}\right)$	1A	
$=\pi^2\left[2r^2h(-2)+(2rh^2+4r^3)\left(\frac{r}{h}\right)\right]$ (by (a))	1M	For sub. $\frac{\mathrm{d}h}{\mathrm{d}t}$ and $\frac{\mathrm{d}r}{\mathrm{d}t}$
$=\pi^2\left(-4r^2h+2r^2h+\frac{4r^4}{h}\right)$		
$=\frac{2\pi^2r^2}{h}(2r^2-h^2)$	1	
(ii) When $t=0$, $h_0=1.2r_0$.		
$\therefore \frac{\mathrm{d}}{\mathrm{d}t}(S^2)\Big\rvert_{t=0}=\frac{2\pi^2{r_0}^2}{1.2r}(2{r_0}^2-1.2^2{r_0}^2)$	1M	For substitution
$=\frac{14}{15}\pi^2{r_0}^3$		OR $\frac{175}{324}\pi^2h^3$
>0	1A	Accept ≥ 0
As t increases,		
r increases and h (>0) decreases and therefore $(2r^2-h^2)$ increases.	1M	
Hence $\frac{\mathrm{d}}{\mathrm{d}t}(S^2)$ increases for all $t\geq 0$.		
i.e. $\frac{\mathrm{d}}{\mathrm{d}t}(S^2)>0$ for all $t\geq 0$		Cannot be skipped
$\therefore \frac{\mathrm{d}S}{\mathrm{d}t}>0$ when $t\geq 0$		
The gatekeeper's claim is agreed.	1	Follow through
	(8)	

Candidates' Performance

Section A (Compulsory)

Question Number		Performance in General
1		Very Good
2	(a)	Very Good
	(b)	Very Good
3		Very Poor
4		Poor
5		Very Good
6		Good
7		Fair
8		Fair
9	(a)	Good
	(b)	Very Poor
10		Good
11	(a)	Good
	(b)	Very Poor
12	(a)	Fair
	(b)	Very Poor
13	(a)	Good
	(b)	Fair

Section B (A choice of 4 out of 5 questions)

Question Number			Popularity (%)	Performance in General
14	(a)		66	good
	(b)			fair
	(c)	(i)		fair
		(ii)		poor
15	(a)		89	very good
	(b)			good
	(c)			fair
16	(a)		85	good
	(b)	(i)		fair
		(ii)		good
	(c)			very poor
17	(a)	(i)	87	good
		(ii)		good
	(b)	(i)		fair
		(ii)		fair
18	(a)		73	good
	(b)	(i)		fair
		(ii)		very poor

Candidates' performance on individual questions

Q.1 Some candidates forgot to write the integration constant and some could not apply the correct formulas. The following mistake was commonly found:

$$\int (8x+5)^{250}\,\mathrm{d}x = \frac{(8x+5)^{251}}{8} + C .$$

Q.2 Some candidates got wrong coefficients or indices in the expansion. In expanding the expression in part (b), a few missed the "..." notation to denote other irrelevant terms.

Q.3 Many candidates could correctly get the intuitive relationship between 22.5° and 45° but could not proceed further. For those who could set up a quadratic equation in obtaining the answer, some forgot to reject the negative root.

Q.4 Most candidates could give $\Delta < 0$ but failed to note the curve was concave downward that implied $k < 0$. Also, many candidates did not solve $k^2 > \frac{1}{36}$ correctly, the following mistake was very common:

$k^2 > \frac{1}{36}$ implies $k > \pm\frac{1}{6}$

Q.5 Many wrong and ambiguous statements were found in the candidates' working:

- "let $n = 1$ be true";
- not defining the statement as $S(n)$ but employing this notation throughout the working;
- in the second step, "Assume the statement is true for <u>all</u> positive integers";
- not stating "the statement is true for $n = 1$" and/or "the statement is also true for $n = k+1$" after finishing the first and/or second steps;
- the word "true" was misspelled as "ture"

Some candidates skipped the essential step(s) in the proof such as writing directly from $\frac{1}{4}k^2(k+1)^2 + (k+1)^3$ to $\frac{1}{4}(k+1)^2(k+2)^2$ without showing the appropriate grouping of terms and factorisation. In this case, the proof would be treated as incomplete.

Q.6 Some candidates could not apply quotient rule correctly. The following mistake was very common:

$(\frac{u}{v})' = \frac{uv'+vu'}{v^2}$ (addition instead of subtraction in the numerator).

Some made mistakes in the simplification of the expression and the substitution. Most could use the correct point-slope form in finding the equation of the tangent.

Q.7 Many candidates did not know the meaning of unit vector. Some stopped only after $|\overrightarrow{OP}|$ was found. Among them, a few claimed $|\overrightarrow{OP}|$ was the answer for unit vector.

Q.8 Most candidates could write down the equation using the area formula. But in expanding the expression, many forgot that there should have ± values. Without realisation of two equations and two possible points, then followed by elimination, marks were not given though the correct answer was obtained. Some candidates noted that "P lies on the second quadrant" and let P be $(-x, y)$. Most of them were confused by the same variables in the given equation, $2x + y - 3 = 0$ and could not find the correct results.

Q.9 In part (a), candidates lost marks in making the following mistakes:

- no steps were presented;
- the answer as $r\sin(x+\alpha)$ was not explicitly written;
- the angle α was written in radian.

In part (b), nearly all candidates failed to consider the increase and decrease of the function in the given range. Most just considered the extreme values of sine function. Some just substituted the terminal values of x in finding the answers.

Q.10 Some candidates confused the calculation of vertical and horizontal stripes. Hence they obtained the following wrong expressions:

1. $\int_a^b (4 - \frac{1}{\sqrt{x}})dx$ 2. $\int_a^b (2 - \frac{1}{y^2})dy$

Others were wrong in the determination of the subtraction of appropriate areas. Many candidates did not handle $\int \frac{1}{\sqrt{x}} \, dx$ correctly and hence got the wrong result.

Q.11 In part (a), a few candidates overlooked the condition that $x \le 3$ and gave steps as

$|x-3|+3=x$

$(x-3)+3=x$ or $(-x+3)+3=x$.

Then, they were confused by the logical arguments and could not obtain the correct answer.
For those who attempted part (b), many knew that they should consider $x>3$ or $x \ge 3$. When the equation came to $x-3+3=x$, however, a number of candidates wrongly obtained "No solution". And for those correctly obtained "all real x", a number of candidates ignored the range set in this case.

Q.12 In part (a), quite a number of candidates could not handle the substitution carefully and a few forgot to simplify the answers.
In part (b) only a few candidates knew the process of finding equation of locus by eliminating k from either

1. the coordinates of P obtained in (a), or
2. the equations L_1 and L_2.

A few candidates obtained the answer as $x^2y+y^3-y^2=0$ or equivalent forms, but forgot to simplify it.

Q.13 In part (a) some candidates found $\frac{dy}{dx}=3x^2-24x+36$, and wrongly simplified this result as $x^2-8x+12$ when equating $\frac{dy}{dx}=0$. Hence they wrongly got $\frac{d^2y}{dx^2}$ and lost marks.
In part (b), quite a number of candidates just sketched the curve up to origin. Some missed the labelling of crucial points, axes and origin. Some could not sketch a smooth curve. A few presented the curve as sinusoidal.

Q.14 (b) Some candidates were unable to eliminate k from the coordinates of the centre $\left(-k+5, \frac{-k}{2}\right)$ to obtain the required locus.

(c) (i) Many candidates did not realise that the point Q was the intersection of $y+5=0$ and the locus in (b).

(ii) Most candidates could not find the slope of L_2. In general, candidates who could give a rough sketch of the circles performed better in this part.

Q.15 (a) Some candidates wrongly assumed that $|\mathbf{p}|=4$ & $|\mathbf{q}|=3$.

(b) Many candidates did not realise that proving any two of OG, AG or BG to be altitudes was sufficient in proving G to be the orthocentre.

(c) Many candidates were unable to apply $\overrightarrow{HN} \cdot \overrightarrow{OA} = 0$ to evaluate t.

Q.16 (b) (i) Some candidates attempted to prove $\angle VDA = 90°$ by assuming $\angle VDA = 90°$ as the first step.

(c) Many candidates thought that the required height was CF, where F was a point lying on VD.

Q.17 (a) (ii) Some candidates wrongly assumed that $\int x \sin x dx = x \int \sin x dx$.

(b) (i) Many candidates confused the two expressions $x^2 \sin x$ and $[x^2 \sin x]_0^{\pi}$.

Q.18 (a) Some candidates did not realise that both r and h were functions of t.

(b) (ii) Many candidates overlooked that the condition $h \doteq 1.2r$ was only valid when $t=0$. This part was more demanding and only a few candidates could give a complete logical explanation.

General comments and recommendations:

1. Candidates should learn to tackle simple problem without guided hint (like Question 3). Stereotyped guided questions might hinder creativity in problem solving skill.

2. Candidates should pay attention to the restriction of the variables specified in the questions. When solving an equation, extraneous solutions should be reasonably rejected. Also, in considering the extreme values of the trigonometric functions, instead of the simplistic quoting of the greatest and smallest values of a sinusoidal function without considering the specified range, the values occurred at the end-points and the turning points in the specified range must be checked.

3. Candidates should clearly show all working steps and provide necessary explanations in order to present a complete and logical solution. Inappropriate practice like rote memorisation or omitting essential steps should be avoided.

4. Candidates should avoid the common errors in the removal of bracket, such as $(2x)^2 = 2x^2$; $\frac{3(x^2+2)-(2x)(3x)}{(x^2+2)^2} = \frac{3x^2+2-6x^2}{(x^2+2)^2}$.

5. The measure of the angles in the steps and the answer, whether in degree or radian, should be aligned with the question.

6. Candidates should use arrow-signs or proper symbols to present vectors in their working. Improper presentation such as $\frac{\overrightarrow{PB}}{\overrightarrow{AP}} = 2$ should be avoided.

7. Candidates should be more proficient in perceiving the graph of a quadratic function and have a better understanding about the relationship between concavity of the curve and the sign of the leading coefficient of the function.

8. Candidates should practise more in $\int x^n \mathrm{d}x$, where n was negative and/or fractional.

9. Candidates should pay attention to the wording and presentation in mathematical induction.

10. Candidates should improve their skills in handling structured questions. Many were unable to relate different parts of the same question.

11. In tackling problems that involved coordinate geometry, candidates performed less favourably when no figure was given in the question. This reflected their lack of basic understanding of actual geometric situations.

12. Candidates should understand thoroughly the presentation of vectors. Mistakes such as $\mathbf{p}^2$, $\frac{\overrightarrow{HM}}{\overrightarrow{OG}} = t$, using $\mathbf{p} \times \mathbf{q}$ to represent $\mathbf{p} \cdot \mathbf{q}$ should be avoided.